



On Radiative Transfer and Gray Body Parameter for Partially Transparent Media

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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ABSTRACT

We obtained a solution to radiative heat transfer equations, which describes a multi-layer gray semi-transparent medium in terms of an opaque body. It is shown that the thermal disturbance between the layers of a medium qualitatively changes its optical properties. We introduced a gray body parameter, which allowed us to describe multilayer gray semi-transparent media with low thermal conductivity. We propose a method for calculating heat transfer by thermal radiation in a gray semi-transparent medium and present the results of radiative heat transfer calculations for screen-vacuum and powder insulation materials as examples of such media.

Keywords: Semi-transparent medium; radiative heat transfer; generalized blackness coefficient; screen-vacuum thermal insulation.

1. INTRODUCTION

The question of radiative heat transfer in a semitransparent medium is of interest both for fundamental physics and for its applications [1-3]. As an example of a semi-translucent medium,

we can give a screen-vacuum insulation, which is a multi-layer granular structure [4]. Radiative heat transfer in such medium determines its heat-insulating properties, to which, at present, high demands are made. While investigating and analyzing opaque medium, radiative heat

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transfer in them is described in terms of models using blackbody coefficients [5]. The existence of a gray body parameter for partially transparent media (which is similar to the blackness coefficient) seems quite natural. Indeed, the Kirchhoff's model, which characterizes the radiation of non-transparent media, should have an analogue in the contiguous class of semi-transparent media.

However, in classical monographs on the theory of radiative heat transfer (for example, in [5-9]), the formalism for calculating such gray body parameter of a semi-transparent body is not given. The authors of these monographs refer to the applicability of Kirchhoff's law for semi-transparent bodies; however, in monographs, different limiting values of the blackness coefficient for a transparent medium are presented. For example, in Ref. [5], this limit is zero due to the absence of radiation and absorption of radiation. However, in Ref. [6], it is noted that when determining the emissivity, data are required that are emitted by the surface of opaque bodies into the half-space. Note that the radiation of semi-transparent bodies is available for observation in a full solid angle; therefore, to preserve the Kirchhoff's law, the upper limit of the value of the blackness coefficient must be 2.

In applied research, radiative heat transfer in opaque optical objects is described using blackness coefficients. In semi-transparent media, radiative heat transfer is described by absorption, transmission, and reflection coefficients. As a result of the presence of a large number of parameters, it is required to solve a bulky system of nonlinear equations, when calculating the heat transfer of multilayer diathermic media. (The numerical solution of such systems is presented, for example, in [10].)

In this work, a non-uniform diametric medium (multilayer plate) is considered under weak nonequilibrium conditions, when the temperatures of the various outer layers of the plate do not differ much. Thermal radiation falls on a plate and the plate is a gray body that absorbs (and emits) radiation at all frequencies. We consider the case of an optically thin medium, in which the effects of self-absorption can be neglected. On the basis of the Stefan-Boltzmann and Kirchhoff equations, we formulated a model of radiative heat transfer in such diathermic environments. We calculated a parameter of the gray body, ε , which characterizes the integral (at all frequencies) emissivity of the medium under weak nonequilibrium. (The gray body parameter ε is similar to the blackness coefficient of the gray body, which characterizes the integral emissivity of the gray body in equilibrium conditions.) The proposed method for calculating the radiative heat exchange of multilayer diathermic media is macroscopic, and does not require a complex analysis of the interaction between radiation and matter at the microphysical level.

2. RADIATIVE HEAT EXCHANGE WITH AN OPAQUE BODY

Let's consider a radiant flow balance on the border of a optically thin gray body (Fig. 1). The incident heat flow Q_{abs} , which is absorbed by the surface of the body with absorptance ε , is returned to the system in the form of thermal return radiation Q_{rad} , and the flow $Q = Q_{rad} - Q_{abs} = Q_{out} - Q_{in}$ is transferred across the boundary in the direction of the outward normal \vec{n} . The return radiation flow Q_{out} is

$$Q_{out} = Q + Q_{in} = Q + \frac{1}{\varepsilon} Q_{abs} = Q + \frac{1}{\varepsilon} (Q_{rad} - Q) = \frac{1}{\varepsilon} Q_{rad} - \frac{1-\varepsilon}{\varepsilon} Q$$

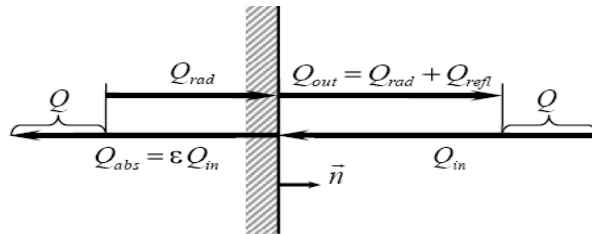


Fig. 1. A radiation flow balance on the border of a gray body: $Q = Q_{rad} - Q_{abs} = Q_{out} - Q_{in}$ Here, Q_{rad} is thermal return radiation, $Q_{abs} = \varepsilon Q_{in}$ is an incident heat flow, ε is absorptance, $Q_{abs} = \varepsilon Q_{in}$ is an absorbed radiation flow, Q_{out} is a return radiation flow

For a system of two parallel surfaces 1 and 2 with an area $S_1 = S_2 = S$, the radiative heat transfer is given by conditions

$$\begin{aligned} Q_{out}^{(1)} &= \sigma S T_1^4 - M_1 Q = Q_{in}^{(2)} \\ Q_{out}^{(2)} &= \sigma S T_2^4 + M_2 Q = Q_{in}^{(1)}, \end{aligned} \quad (1)$$

where $M_{1,2} = 1/\varepsilon_{1,2} - 1$, T_1 and T_2 are temperatures at surfaces 1 and 2, respectively.

In (1), σ is the Stefan-Boltzmann constant, $\sigma = 5.67 \cdot 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$.

Note that the dependence of the density of blackbody radiation on temperature as the fourth degree of temperature was formally derived for conditions of equilibrium radiation [11-12]. However, it is shown that this law can also be used for the case of weakly nonequilibrium systems (see, for example, [13-14] and references in them). This law is widely used to study the energy transfer by radiation inside stars and in their surface layers, as well as in solving various issues relating to the interaction of electromagnetic radiation with the matter of cosmic objects [15-16]. However, this law cannot be applied, for example, to the solar corona, in which the conditions of thermodynamic equilibrium are strongly violated. This law is also unsuitable for determining the emissivity of non-thermal sources, such as laser sources. In this paper, we investigate a weakly nonequilibrium system.

Since the flow from the surface 1 to 2 is equal to $Q = Q_{out}^{(1)} - Q_{in}^{(1)} = Q_{in}^{(2)} - Q_{out}^{(2)}$, we have

$$Q = \frac{1}{M_1 + M_2 + 1} \sigma S (T_1^4 - T_2^4) = \frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2} \sigma S (T_1^4 - T_2^4) \quad (2)$$

Formula (2) can be obtained by considering the processes in which radiation quanta are created/absorbed at the body boundaries 1 and 2 with probabilities ε_1 and ε_2 , respectively. The product $P = \varepsilon_1 \varepsilon_2$ is the heat transfer probability, and the value of $R = 1 - (1 - \varepsilon_1)(1 - \varepsilon_2) = \varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2$ determines the probability of quantum annihilation under reflection. Then the ratio P/R is the probability of heat transfer taking into account reflections.

Let's deduce the basic equation (1) in another way. Let the ε -th part of the surface S be blackbody surfaces, and the $(1 - \varepsilon)$ -th part be the ideal reflecting areas. The total return flow Q_{out} of such a mosaic surface is a superposition of the flows of all its elements:

$$Q_{out} = \varepsilon S \sigma T^4 + (1 - \varepsilon) Q_{in} = \varepsilon S \sigma T^4 + (1 - \varepsilon)(Q_{out} - Q)$$

By regrouping the terms of this expression, we obtain the equation (1). Thus, a mosaic, consisting of regions with different optical properties, behaves on average like a gray body (it follows from Huygens' equivalence principle). For such a macroscopic description, it is necessary to estimate the uniformity of the flow Q_{in} in the region of the conjugate screen. For a large number of mosaic elements, the mixture of outgoing flows occurs due to a wide spectrum, as well as non-directionality and incoherence of thermal radiation. In the case when inhomogeneity of flow Q_{in} cannot be neglected, it is necessary to consider separately the heat exchange in the spatial, angular or spectral zones.

In the general case, in the space between surfaces with different temperatures, the radiation is non-equilibrium and anisotropic. By averaging the radiant flows over the solid angle 4π , we obtain the expression for the brightness temperature θ as the temperature of a homogeneous medium, including a semi-transparent test medium (for example, a gas), which is in equilibrium with the radiation. We have

$$\theta^4 = \frac{Q_{out}^{(1)} + Q_{out}^{(2)}}{2\sigma S} = \frac{T_1^4 + T_2^4}{2} + \frac{M_2 - M_1}{2\sigma S} Q \quad (3)$$

Substituting the flow Q from the expression (2) into (3), we obtain

$$\theta^4 = \frac{T_1^4 + T_2^4}{2} + \frac{M_2 - M_1}{M_1 + M_2 + 1} \frac{T_1^4 - T_2^4}{2} \quad (4)$$

Formula (4) allows us to verify the correctness of the calculation of radiative heat transfer in both opaque and diathermic media. We note a remarkable property of brightness temperature. Using expressions (2), (3), we get the following simple expressions:

$$\begin{aligned} T_1^4 - \theta^4 &= \frac{M_1 + 1/2}{\sigma S} Q \\ \theta^4 - T_2^4 &= \frac{M_2 + 1/2}{\sigma S} Q \end{aligned} \quad (5)$$

From (5) it follows that the radiative heat exchange between the bodies is reduced to a separate heat exchange of each of the bodies with radiation in the cavity adjacent to this body. The parameter $(M + 1/2)/\sigma S$ has the meaning of the thermal resistance attached to the screen, and the thermal resistance value is determined by the screen parameters. In complex systems, it is possible to use the method of equivalent replacement of layers and zones by a series-parallel connection of their thermal resistances.

We now consider the one-dimensional problem of radiative transfer by a system of bodies. As an example, we define a gray body parameter ε_1 of a single screen, which is equivalent to a group of k identical screens ($k = 1, 2, \dots$) with thermal emissivity ε_k .

From (5) it follows that the difference in the values of $\theta_A^4 - \theta_B^4 = Q(2M + 1)/(\sigma S)$ of the brightness temperature of radiation on different sides (A and B) of the screen is determined by the value of the through flow Q (the flow Q is common to all layers). Thus, we have the relation $2M_1 + 1 = k [(1 - 2\varepsilon_k)/\varepsilon_k + 1]$. Then

$$\varepsilon_1 = \frac{2\varepsilon_k}{k(1 - \varepsilon_k) + \varepsilon_k}, \quad \lim_{k \rightarrow \infty} \varepsilon_1 = 0. \quad (6)$$

Inversion (6) allows us to solve the inverse problem of equivalent replacement of a single screen with a grayness parameter ε_1 by a group of identical screens:

$$\varepsilon_k = \frac{k\varepsilon_1}{(k - 1)\varepsilon_1 + 2}, \quad \lim_{k \rightarrow \infty} \varepsilon_k = 1. \quad (7)$$

The grayness parameter ε_1 has the meaning of thermal emissivity in the problem under consideration. (We consider the body under weak non equilibrium conditions.) The parameter ε is similar to the blackness coefficient of the gray body, which characterizes the integral

emissivity of the gray body in equilibrium conditions.

Thus, an increase in the number of layers (screens) approximates the optical characteristics of the group to the properties of an ideal mirror with a high degree of accuracy. However, inverse operation has a limitation. The group, which replaces the screen with a gray body parameter ε_1 , may contain no more than $k = 2/\varepsilon_1 - 1$ layers, since $\varepsilon \leq 1$. In particular, a completely black screen cannot be replaced by anything else.

3. RADIATIVE TRANSFER IN DIATHERMIC BODIES

Consider the radiative transfer in a flat diathermic layer. (Diathermy is a property of partial transmission of thermal radiation. In qualitative descriptions, the term diathermy is synonymous with semi-transparent. In quantitative calculations, the term diathermy is used as the equivalent of the transmittance.) Following the example of the previous section, we construct the equivalent of a semi-transparent body with optical characteristics given by the absorptance A , reflectance R , and transmittance D . ($A + R + D = 1$) Let's fill the surface of the equivalent screen with a mosaic of transparent and opaque fragments with the ratio of areas of $D/(D - 1)$. To preserve the flux values Q_{abs} and Q_{refl} , we choose the coefficient A^* and R^* of the solid part of the surface as

$$\begin{aligned} \varepsilon^* &= A^* = A/(1 - D), \quad R^* = R/(1 - D), \\ A^* + R^* &= 1 \end{aligned} \quad (8)$$

Let's choose a positive direction along the outward normal vector, and write the expression for outgoing flows $Q_{out}^{(A)}$ and $Q_{out}^{(B)}$ in the form of superpositions of return flows of opaque areas and transit flows of transparent elements:

$$\begin{aligned} Q_{out}^{(A)} &= (1 - D)(\sigma ST^4 - M^*Q) + D Q_{in}^{(B)} \\ Q_{out}^{(B)} &= (1 - D)(\sigma ST^4 - M^*Q) + D Q_{in}^{(A)}, \end{aligned} \quad (9)$$

where $M^* = (1 - \varepsilon^*)/\varepsilon^* = R^*/A^* = R/A$.

Substitution of $Q_{in}^{(A)} = Q_{out}^{(A)} - Q$ and $Q_{in}^{(B)} = Q_{out}^{(B)} - Q$ in (9) allows us to obtain two

independent equations for the flows emitted from each side of the screen:

$$\begin{aligned} Q_{out}^{(A)} &= (1-D)(\sigma ST^4 - M^*Q) + D[(1-D)(\sigma ST^4 - M^*Q) + D(Q_{out}^{(A)} - Q) - Q] \\ Q_{out}^{(B)} &= (1-D)(\sigma ST^4 - M^*Q) + D[(1-D)(\sigma ST^4 - M^*Q) + D(Q_{out}^{(B)} - Q) - Q] \end{aligned} \quad (10)$$

For a transparent body ($D=1$), relations (10) become identities. In other cases, the equations are reduced to the form of the type (1):

$$\begin{aligned} Q_{out}^{(A)} &= \sigma ST^4 - \tilde{M} Q \\ Q_{out}^{(B)} &= \sigma ST^4 + \tilde{M} Q \end{aligned} \quad (11)$$

where

$$\tilde{M} = \frac{(1-D)M^* - D}{1+D}. \quad (12)$$

So, we have described the semi-transparent screen as an opaque body. In deriving the formulas, we used only the Huygens's principle and the classical formulation of the Kirchhoff's law on the equality of absorptivity and emissivity for an opaque body. The identity of the descriptions (1) and (11) allows us to define a gray body parameter $\tilde{\varepsilon}$ and coefficient $\tilde{R} = 1 - \tilde{\varepsilon}$ for an diathermic medium. The definition is based on the characteristic features of an opaque body, which is thermophysical equivalent of diathermic body.

By analogy with (7), we obtain the required expressions for the gray body parameter:

$$\tilde{\varepsilon}(A, D) = \frac{A(1+D)}{2(1-2D+AD+D^2)}, \quad (13)$$

Fig. 2 shows series of dependencies for the gray body parameters $\tilde{\varepsilon}(A, D)$.

As an illustration, we present the calculations of radiation heat transfer in a sample of screen-vacuum thermal insulation, which is located between the massive plates of the heater and the refrigerator. The calculation was carried out for a sample consisting of 19 isolated semi-transparent screens with a blackness coefficient of $\varepsilon^* = 0.04$ (Fig. 3).

For the temperature $T_1 = 573K$ at the cold boundary and the heat flow density $\dot{Q} = 1 W/m^2$, we determine the temperature of the intermediate layers and temperature T_2 at the hot boundaries, as well as the radiant temperature θ for the adjacent layers. We used the temperatures T_1 and T_2 to calculate the control value of the brightness temperature θ^* in the model without screen-vacuum thermal insulation. Fig. 3 shows the results of our calculations in the form of bar charts.

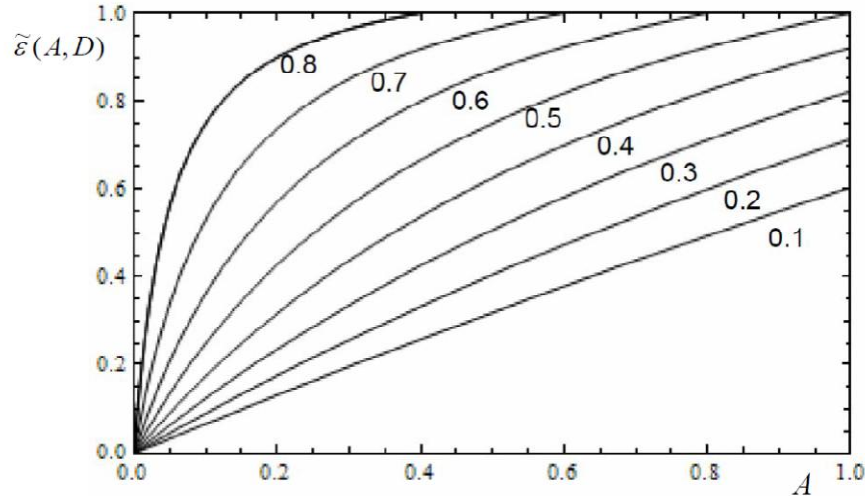


Fig. 2. Dependencies of the gray body parameters $\tilde{\varepsilon}(A, D)$ on absorptance A for different values of transmittance D (the values of transmittance D are shown below the curve)

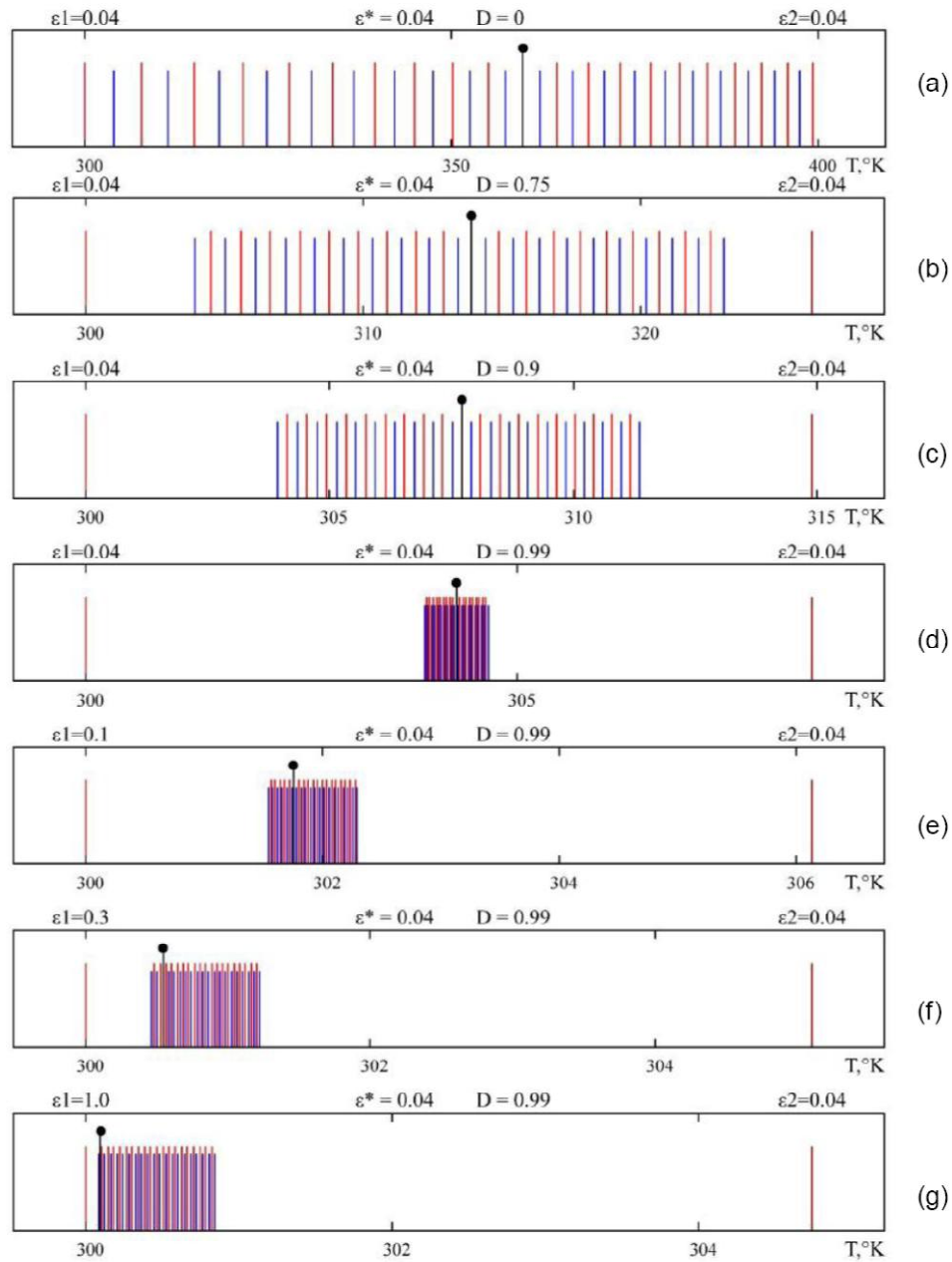


Fig. 3. The results of the heat transfer calculations in samples of screen-vacuum thermal insulation. The top line shows the parameters of the borders ($\varepsilon_1, \varepsilon_2$) and layers (ε^*, D); the solid red line is the layer temperature, the blue line is the brightness temperature θ ; a line with a circle on top is results of the heat transfer calculations in a model without screen-vacuum thermal insulation

The increase in transparency of the screen-vacuum thermal insulation in the range of $D=0...0.99$ (Fig. 3, a - d) leads to a decrease in the temperature difference $T_1 - T_2$. The

diathermic layers have temperatures close to the check value θ^* , and the diathermic layers distort heat flows less and less, gradually turning into test bodies. As the value of ε_1 of the left border

increases from $\varepsilon_1 = 0.04$ to 1 (Fig. 3, d - g), the position of the brightness temperature θ^* is shifted. The diathermic group repeats the same characteristic changes.

Thus, the representation of a system of bodies by a chain of equivalent radiation resistances correctly describes characteristic features of radiative heat transfer in media, which consists of alternating heat-conducting and heat-insulating layers. Well-studied examples of such composite materials are screen-vacuum and powder thermal insulation [17]. We believe that the approach presented here allows us to extend the application area of classical theory to the class of semi-transparent media.

In the considered problem of the equivalence of layers, the introduction of the gray body parameter removes the restriction on the number of layers, and allows us to proceed to the description of continuous media with low thermal conductivity. By substituting (13) into (5), we obtained a difference in brightness temperature $\theta_A^4 - \theta_B^4$ at the boundaries of the layer for the sum of radiation resistances with an unlimited increase in their number:

$$\theta_A^4 - \theta_B^4 = \frac{2\tilde{M}+1}{\sigma S} Q = \frac{2M^*+1}{\sigma S} \cdot \frac{1-D}{1+D} Q \quad (14)$$

Note that in order to substantiate the proposed approach, it is necessary to investigate heat exchange processes in heat-conducting media, since the conclusions of Sections 2 and 3 implicitly imply the fulfillment of an isothermal condition $T_A = T_B = T$ during the measurement of optical parameters.

4. KIRCHHOFF'S LAW FOR SEMI-TRANSPARENT MEDIA

By proposing the equality of the emissivity and absorption capacity of bodies, the Kirchhoff law excludes the possibility of a circular Thomson-Planck process, that is, it excludes the possibility of creating a second-order perpetual motion machine. In this regard, it is puzzling the statement (for example, in [18]) that the Kirchhoff law is valid only for cases of thermal equilibrium.

Within the Kirchhoff's model, there is a consideration of the process of mixed heat exchange in heat-conducting optical materials

consisting of contacting opaque layers. From equation (5) it follows that the equilibrium and isotropic radiation at the internal boundaries has a brightness temperature equal to the thermodynamic temperature of the body. In this case, the emissivity for a group of layers is completely determined by the optical properties of the outer surface.

In fact, every physical body spontaneously and continuously emits electromagnetic radiation. The spectral radiance of the body depends on the temperature and frequency of the radiation and describes the amount of energy it emits at different radiation frequencies. In the limit of strong absorption (In the limit of a black body) near thermodynamic equilibrium, the spectral radiance is closely described by Planck's law [19-20]:

$$B(\nu, T) = \frac{2\pi\nu^3}{c^2} \frac{1}{e^{h\nu/k_B T} - 1}$$

where $B(\nu, T)$ is the spectral radiance (the power per unit solid angle and per unit of area normal to the propagation) density of frequency ν radiation per unit frequency, k_B is the Boltzmann constant, h is the Planck constant, and c is the speed of light in the medium. By integrating $B(\nu, T)$ over the frequency and by subsequently integrating over the solid angle, the Stefan-Boltzmann law is calculated, stating that the power emitted per unit area of the surface of a black body is directly proportional to the fourth power of its absolute temperature [12]. Kirchhoff has formulated the law of thermal radiation, which states for an arbitrary body emitting and absorbing thermal radiation in thermodynamic equilibrium, the emissivity is equal to the absorptivity [11].

It would seem that the same reasoning can be applied to a weak non equilibrium system of coupled semi-transparent layers with an arbitrary value of emissivity. However, if this were true, then we would not be able to see the objects of the outside world. This paradoxical conclusion follows from Kirchhoff's law, which does not allow radiative heat transfer between bodies with equal temperature. Thus, the Kirchhoff's law in its classical formulation is not satisfied in semi-transparent media under weak non equilibrium conditions.

The radiative heat process discussed above does not change the thermodynamic state of the body, and, therefore, the process must be considered separately from energy-dependent emission and absorption processes. We calculate a parameter of the gray body, which characterizes the integral (at all frequencies) emissivity of the medium under weak non equilibrium. The gray body parameter is similar to the blackness coefficient of the gray body, which characterizes the integral emissivity of the gray body in equilibrium conditions.

5. A GENERAL EQUATION OF RADIATIVE HEAT TRANSFER

Let $Q^{(A)}$ and $Q^{(B)}$ be the resultant flows in the direction of the outward normal to the screen surfaces A and B , respectively. Let us write expressions for outgoing flows, $Q_{out}^{(A)}$ and $Q_{out}^{(B)}$, as a superposition of return flows of gray areas and transit flows of transparent elements:

$$\begin{aligned} Q_{out}^{(A)} &= (1-D)(\sigma ST^4 - \tilde{M}Q^{(A)}) + D Q_{in}^{(B)} \\ Q_{out}^{(B)} &= (1-D)(\sigma ST^4 - \tilde{M}Q^{(B)}) + D Q_{in}^{(A)} \end{aligned} \quad (15)$$

Taking into account $Q_{in}^{(A)} = Q_{out}^{(A)} - Q^{(A)}$ and $Q_{in}^{(B)} = Q_{out}^{(B)} - Q^{(B)}$, the system of equations (15) is reduced to two independent equations for the flows outgoing from each side of the screen:

$$\begin{aligned} Q_{out}^{(A)} &= (1-D)(\sigma ST^4 - \tilde{M}Q^{(A)}) + D[(1-D)(\sigma ST^4 - \tilde{M}Q^{(B)}) + D(Q_{out}^{(A)} - Q^{(A)}) - Q^{(B)}] \\ Q_{out}^{(B)} &= (1-D)(\sigma ST^4 - \tilde{M}Q^{(B)}) + D[(1-D)(\sigma ST^4 - \tilde{M}Q^{(A)}) + D(Q_{out}^{(B)} - Q^{(B)}) - Q^{(A)}] \end{aligned} \quad (16)$$

To clarify the physical meaning of expression (16), we represent the resulting flows in (16) as a

superposition of symmetric and antisymmetric flows:

$$\begin{aligned} Q^{(A)} &= Q_s + Q_a, \text{ where } Q_s = (Q^{(A)} + Q^{(B)})/2 \\ Q^{(B)} &= Q_s - Q_a, \text{ where } Q_a = (Q^{(A)} - Q^{(B)})/2. \end{aligned} \quad (17)$$

Using simple transformations (17), we obtain

$$\begin{aligned} Q_{out}^{(A)} &= \sigma ST^4 - M_s Q_s - M_a Q_a, \\ Q_{out}^{(B)} &= \sigma ST^4 - M_s Q_s + M_a Q_a \end{aligned} \quad (18)$$

where

$$M_s = \frac{(1-D)\tilde{M} + D}{1-D}, \quad M_a = \frac{(1-D)\tilde{M} + D}{1+D}.$$

A comparison of (1) and (18) shows that for both symmetric ($Q_a = 0$) and antisymmetric ($Q_s = 0$) heat exchange, the semi-transparent screen can be considered as an opaque body. Identity (1) and (18) allows us to determine the gray body parameter of a diathermic medium based on the characteristics of its opaque equivalent

$$\begin{aligned} \tilde{\varepsilon}_s(A, D) &= \frac{A(1-D)}{2(1-2D+AD+D^2)}, \\ \tilde{\varepsilon}_a(A, D) &= \frac{A(1+D)}{2(1-2D+AD+D^2)} \end{aligned} \quad (19)$$

From (19) it follows that the gray body parameters ε_s and ε_a do not coincide between themselves, and this fact is an essential feature of diathermic media. With increasing transparency, the values of the gray body parameter ε_s tend to zero and the value of ε_a approaches the limit value 1.

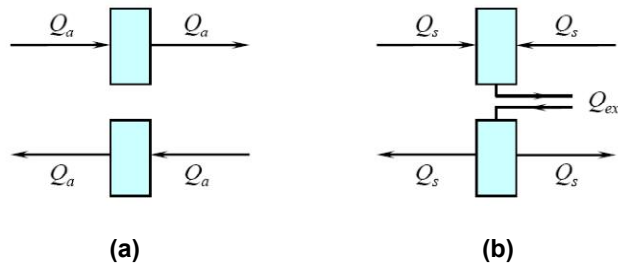


Fig. 4. Schematic picture of resulting flows Q_a and Q_s in antisymmetric (a) and symmetric (b) structures, respectively. External heat flows Q_{ext} is converted from radiant flows

Indeed, in the case of antisymmetric resulting flows (Fig. 4a), there is no conversion of radiant flows into thermal flows. But in the case of symmetric resulting flows (Fig. 4b), the radiant flows are completely converted into thermal flows.

Now let's consider a model in which we replace a semi-transparent medium with an equivalent quadrupole consisting of radiation and thermal resistances. Let's set the values $Q^{(A)}$ and $Q^{(B)}$ of thermodynamic temperature T_A and T_B of the medium and for each side of the translucent layer. Note that despite the four-way flow exchange, equation (11) contains only two linearly independent variables due to two constraints,

$$Q_{ext}^{(A)} + Q_{ext}^{(B)} = -2Q_S,$$

$$Q^{(A)} + Q^{(B)} = 2Q_S,$$

prescribed by the Energy Conservation Law.

Below, we will denote the coefficient at the resulting radiant flow in Eq. (5) by the symbol R (this coefficient has the meaning of radiation resistance). Denote also the coefficient at the thermal flow in the thermal conductivity equation

$T_A - T_B = rQ$ by the symbol r (r has the meaning of thermal resistance).

The black-body concept, introduced by Kirchoff in 1860 [11], assumes that the near-surface layer is infinitely thin and ideally absorbs radiation. Later, Planck pointed to the internal contradictions of the concept, noting that a) the radiation should penetrate into the body, but the radiation should not be reflected; b) the body should have a minimum thickness sufficient to absorb the incident flow; c) there should be strict restrictions on the scattering of radiation in the body [19-20].

Using the operations of splitting and merging layers, we find $\tilde{\varepsilon}_0$ and $\tilde{\varepsilon}_1$ for rational values of $k > 0$. Using $\tilde{R} = 1 - \tilde{\varepsilon}$, we have

$$\tilde{R} = \frac{k(\tilde{R}_k + 1) + \tilde{R}_k - 1}{k(\tilde{R}_k + 1) - \tilde{R}_k + 1}. \quad (20)$$

The expression (20) can be written in a simpler form if we take the number of layers K , which is equivalent to a black screen ($\tilde{R} = 0, \tilde{\varepsilon} = 1$), as a unit of diathermal thickness. Then, we have

$$\tilde{R}(x) = \frac{x-1}{x+1}, \text{ where } x = \frac{k}{K}, \text{ and } K = \frac{\tilde{R}_1 - 1}{\tilde{R}_1 + 1}. \quad (21)$$

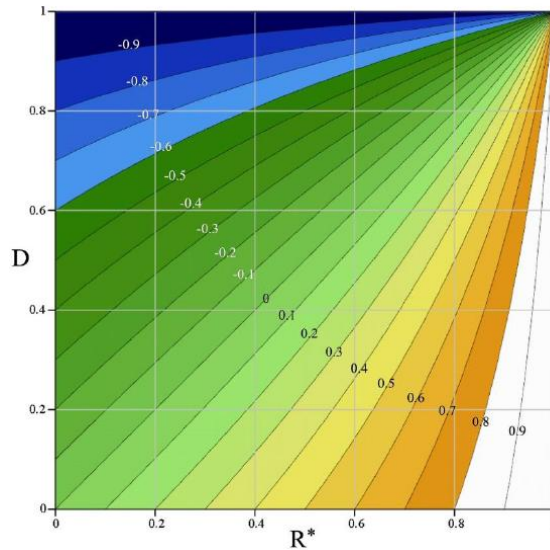


Fig. 5. Isolines of the surface of a coefficient $\tilde{R}(R^*, D)$. Here, $R^* = R/(1-D)$, R is reflectance, D is transmittance

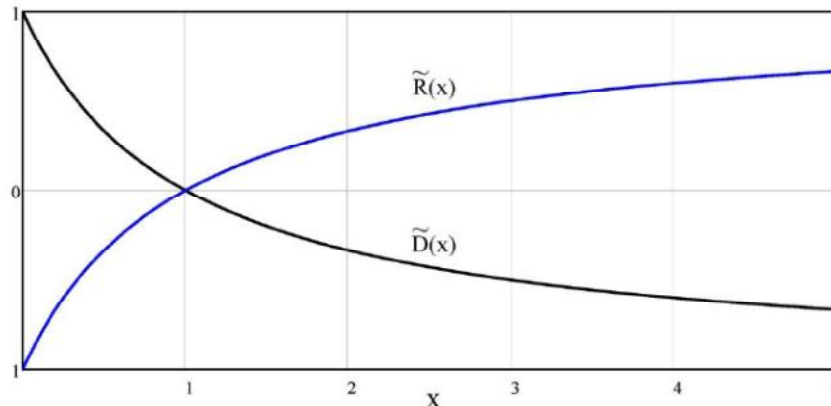


Fig. 6. Dependence of a coefficients $\tilde{R}(x)$ and $\tilde{D}(x)$ on a diathermic layer thickness x

The gray body parameter $\tilde{\varepsilon}$ includes independent optical characteristics, which are due to the different nature of heat transfer mechanisms. In Fig. 5, surface contour lines $\tilde{R}(R^*, D)$ show the contribution of various mechanisms to heat transfer. The absorbed flow is converted into heat by interaction with the substance, and is transferred through the body volume due to heat conduction. According to Kirchhoff's law, on the other side of the body, there is a conversion of heat into radiant energy. This mechanism is typical for bodies with parameter $\tilde{R} > 0$. For diathermic media with parameter $\tilde{R} < 0$, heat transfer occurs mainly due to the difference in transit flows. The isoline $\tilde{R} = 0$ corresponds to the transition from the mode of primary radiation transmission to the mode of radiation reflection. Fig. 6 shows the dependences of coefficients $\tilde{R}(x)$ and $\tilde{D}(x)$ on a diathermic layer thickness x . The function has a symmetric form, and in the conditions of complementary processes, we can write $\tilde{D} = -\tilde{R}$.

The dependences $\tilde{D}(x)$ show that the Bouguer – Lambert – Beer law is not observed in the case of radiative transfer in diathermic systems with a large optical thickness. This is due to the phenomena of multiple reflection and re-radiation (diffusion) of the primary radiant flow.

6. CONCLUSION

In this work, in the framework of the macroscopic approach, we propose a method for calculating radiant heat transfer, which is applicable to both opaque and diathermic weak nonequilibrium media. The universal algorithm for describing

various media is due to the possibility of replacing the optical characteristics of the body (absorptance, reflectance, and transmittance) by a gray body parameter $\tilde{\varepsilon}$, which describes a semi-transparent medium as opaque. It is shown that the representation of a system of partially transparent bodies by a chain of equivalent radiation resistances makes it possible to describe radiative heat transfer in media consisting of intermittent heat-conducting and heat-insulating layers. The introduction of a gray body parameter for semi-transparent layers made it possible to remove the restriction on the number of splitting layers, and thus proceed to the description of continuous media with low thermal conductivity (examples of such media are screen-vacuum and powder thermal insulation). Considered screen-vacuum and powder insulation are examples of such media. The proposed method for calculating heat transfer in a semi-transparent medium can also be used in aeronomic studies of the Earth and planets, as well as in astrophysics.

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COMPETING INTERESTS

Author has declared that no competing interests exist.

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