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Quantum Effects on Rayleigh-Taylor Instability of a Plasma-Vacuum

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Author's contribution

The only author performed the whole research work. Author GAH wrote the first draft of the paper. Author GAH read and approved the final manuscript.

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ABSTRACT

The hydrodynamic instability of unmagnetized quantum plasma layer supported by magnetized vacuum layer is investigated. The plasma is considered as incompressible, inviscid and has exponentially varying density. The relation between square normalized growth rate and square normalized wave number is obtained and analyzed. The results are shown that, the interface is more stability in the presence of quantum effect beside the magnetic field effect.

Keywords: Rayleigh-Taylor instability; quantum plasma; vacuum magnetic.

1. INTRODUCTION

The field of quantum plasmas has been introduced since long ago. Klimontovich and Silin [1] derived a general kinetic equation for quantum plasmas and studied the dispersion properties of electromagnetic waves. Some other developments of that time include the equilibrium theory of quantum plasmas using a procedure similar to Feynman's methods in field theory [2], dielectric formulation of quantum statistics in random phase approximation [3]; and the self-consistent field approach to many-electron problem [4]. For non-equilibrium homogenous systems, kinetic equations have been derived by Balescu [5]. Guernsey [6]

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used an approach originally developed by Bogoliubov to present a unified theory of equilibrium and non-equilibrium quantum plasmas. Pines [7] studied the dynamics of quantum plasmas with particular attention to the relationship between individual particle and collective behavior. A general theory of electromagnetic properties of the electron gas in a quantizing magnetic field was also developed treating the electrons quantum mechanically [8-9].

It is well known, the pure classical plasmas has many application in compact astrophysical objects such as in white dwarfs and the atmosphere of neutron stars [10] or in the next generation intense laser-solid density plasma interaction experiments [11]. But, many-body charged particle systems cannot be treated by pure classical physics when the characteristic dimensions become comparable to the de Broglie wavelength. This is the case for quantum semiconductor devices, like high-electron-mobility transistors, resonant tunneling diodes or super-lattices. The operation of these ultra-small devices relies on quantum tunneling of charge carriers through potential barriers. So, quantum effects in plasma become important in like these environments.

In magnetic fusion research interface problem arise naturally the form the requirement that thermonuclear plasma be confined and isolated from the outside world by a vacuum magnetic field. The plasma-vacuum interface problem in magneto-hydrodynamics was presented by Goedbloed [12]. The Rayleigh-Taylor instability (RTI) of plasma-vacuum boundary with the aid of an ideal one-fluid Hall model corresponding to the limit of a large ion Larmor radius was studied Velikovich [13]. The stability of gravitational compressible surface modes of a plasma-vacuum interface is studied by Grattojn et al. [14] and by Alejandro et al. [15].

In the last years, the hydrodynamic instability on the interface between a vacuum and quantum plasma has attracted much attention because of wide applications in many areas such as laser physics, plasma spectroscopy, plasma technology, and surface science [16-18]. For example, plasma and vacuum technologies are used in the microelectronics, communications, biomedical and other modern manufacturing industries. Vacuum plasma processing is already a well-proven and widely-used technique for etching and surface modification in the electronics industry. It is being increasingly used by the aerospace, automotive, medical, military and packaging industries for cleaning and surface engineering of plastics, rubbers and natural fibers as well as for replacing CFCs for cleaning metal components, polypropylene automotive components such as car bumpers, door mirror housings and dash board components are plasma treated before painting.

The instability of quantum plasma-vacuum interface is studied of many different models [19-23].The vacuum- quantum plasma composed of electron, including the effects of a quantum statistical Fermi electron temperature and the system is acted upon by an electromagnetic field ((E_x, E_y, B_z) TM-model) is studied by Lazar et al. [19]. In this model the initial values for electric and magnetic field were vanished ($E_0 = B_0 = 0,$ Electrostatic model). The same model is considered by Mohamed [20], but in his model the initial value of magnetic field was taken account ($\vec{B}_0 = B_0\, e_y$, transverse magnetic field). The previous model under the effect of electromagnetic field ((B_x, B_y, E_z), TE-model) is studied by Mohamed and Abdel Aziz [21]. The instability of the interface between a quantum magneto-plasma composed of electrons and positrons, and vacuum are studied by Misra et al. [22]. In this study the external magnetic field lies in the $x-z$ plane making an angle θ with the z axis. A quantum surface mode at a plasma-vacuum interface with uniform magnetic field is studied in quantum electron-hole semiconductor plasma by Misra [23]. In the above studies [19-23] the systems had a small-amplitude perturbation, where some terms (higher orders derivatives) of the linearized equations are deleted.

In this paper, the classical RTI model in refs. [12,13] will be again studied in quantum plasmas. The surface of discontinuity ($Z=0$) has considered between infinitely conducting plasma in the half-space $Z<0$ and a vacuum in the other half-space $z>0$,that has been permeated by a uniform horizontal magnetic field ($\vec{B}_0 = B_0\,\vec{e}_x$). Here, we use a system of Cartesian coordinates, where $Z-$axis in the vertical direction. A gravitational acceleration $g=(0,0,g)$ directed from the plasma towards the vacuum. In all the above studies (19-22), the perturbation was very slow. So, the higher derivatives that rise in the system considerable are neglected. In our analysis the perturbation will be superabundant (high-speed), such that the system cannot return to the initial case (i. e. The system will remain in a permanent disturbance case). Thus all the terms, which will rise in the linearized equations, will be considered.

2. GOVERNING EQUATIONS AND LINEAR PERTURBATIONS

For incompressible quantum plasma as a fluid of electrons and immobile ions the relevant equations may be written as (see refs. [19 -26])

$$\left(\frac{\partial}{\partial t}+\vec{\mathrm{U}}\cdot\nabla\right)\rho\,\vec{\mathrm{U}}=-\nabla P+\rho\,\vec{g}+\vec{Q}\,, \tag{1}$$

$$\nabla\cdot\vec{\mathrm{U}}=0\,, \tag{2}$$

$$\left(\frac{\partial}{\partial t}+\vec{\mathrm{U}}\cdot\nabla\right)\rho=0\,, \tag{3}$$

$$\left(\frac{\partial}{\partial t}+u_x\frac{\partial}{\partial x}+u_y\frac{\partial}{\partial y}\right)\eta=u_z\Big|_{\text{at the interface}} \tag{4}$$

Here $\vec{\mathrm{U}}(u_x,u_y,u_z)$ is the velocity, ρ is the density, p thermal pressure, $\vec{g}$ is the gravitational acceleration. A quantum effect in equation (1) is represented by Bohm Potential ($\vec{Q}=\frac{\hbar^2}{2m_e m_i}\rho\nabla\left(\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right)$), m_e is the electron mass and m_i the ion mass. η is the displacement which is a function of x, y and t. The pressure term in Eq. (1) contains both the Fermion P_F, and the thermal P_t. For high temperature plasmas, it is simply thermal pressure P_t. However, for very low temperature plasma by assuming that the ions behave classically in the limit $P_{F_i} << P_{Fe}$ (where P_{F_i} is the ion Fermi temperature and P_{F_e} is the electron Fermi temperature), the pressure effects of quantum electrons are relevant only. In this situation the Fermi pressure which is contribution of the electrons obeying the Fermi-Dirac equilibrium is of most significance. The Fermi pressure increases with increase in number density and is different from thermal pressure ($P_F = \left(\frac{4\pi^2\hbar^2}{5m}\right)\left(\frac{3}{8\pi}\right)^{\frac{2}{3}} n^{\frac{5}{3}}$, n is the number density). In our study, we consider the case of high temperature plasmas, so the thermal pressure ($P_t = P$) plays a vital role.

Now, if we wish to study quantum plasma-vacuum system with an interface need to supplement Eqs. (1)-(4) with the equations describing the vacuum magnetic field $\vec{B}$:

$$\nabla \cdot \vec{B} = 0, \; \nabla \times \vec{B} = 0. \tag{5}$$

These equations are all that is left from Maxwell's equations when the displacement current is negligible. So that $\vec{B} = \nabla\varphi$ may be obtained as the gradient of a scalar φ which satisfies the equations

$$\nabla^2\varphi = 0, \; \nabla \times \nabla\varphi = 0. \tag{6}$$

The dynamic boundary condition across the interface of plasma-vacuum is

$$< P_s > = < \frac{B^2}{\mu} >, \tag{7}$$

the brackets $< >$ is the jump across the interface, P_s the pressure through the two layers and μ is magnetic permeability. The condition on the magnetic field at the interface with a perfect conductor is

$$\vec{n} \cdot \vec{B} = \vec{n} \cdot \nabla\varphi = 0, \tag{8}$$

where $\vec{n}$ is the unit vector which is the outward normal to the surface. This vector is defined by $f = z - \eta(x, y, t)$. According to nonlinear Fourier perturbation and elaborated by Callebaut, where every physical quantity (say X) can be expanded in a series ε:

$$X = X_0 + \varepsilon X_1 + \varepsilon^2 X_2 + \varepsilon^3 X_3 + \ldots\ldots\ldots\ldots\ldots\ldots \tag{9}$$

where ε is the amplitude of the first order term at all times, $\varepsilon = \varepsilon_0 \exp(\omega t)$ and ε_0 is its amplitude at t = 0 and $\omega = -i\sigma$ is the frequency of perturbations or the rate at which the system departs from equilibrium thee initial state (Plasma frequency). Here, in our example of RTI, we consider the plasma layer is initially at rest. This means that $\vec{U}_0 = 0$ and $\eta_0 = 0$.

Then, the linearized equations of quantum plasma layer (by Eqs. (1)- (4)) may be written as

$$\rho_0 \omega \vec{U}_1 = -\nabla P_1 + \rho_1 \vec{g} + \vec{Q}_1, \tag{10}$$

$$\nabla \cdot \vec{U}_1 = 0, \tag{11}$$

$$\omega \rho_1 + (\vec{U}_1 \cdot \nabla) \rho_0 = 0, \tag{12}$$

$$\omega \eta_1 = u_{z1}\big|_{z=0}, \tag{13}$$

where $\vec{Q}_1$ is given by

$$\vec{Q}_1 = \frac{\hbar^2}{2m_e m_i} \left\{ \begin{array}{l} \frac{1}{2}\nabla(\nabla^2\rho_1) - \frac{1}{2\rho_0}\nabla\rho_1\nabla^2\rho_0 - \frac{1}{2\rho_0}\nabla\rho_0\nabla^2\rho_1 + \\ \frac{\rho_1}{2\rho_0^2}\nabla\rho_0\nabla^2\rho_0 - \frac{1}{2\rho_0}\nabla(\nabla\rho_0.\nabla\rho_1) + \frac{\rho_1}{4\rho_0^2}\nabla(\nabla\rho_0)^2 + \\ \frac{1}{2\rho_0^2}(\nabla\rho_0)^2\nabla\rho_1 + \frac{1}{\rho_0^2}(\nabla\rho_0.\nabla\rho_1)\nabla\rho_0 - \frac{\rho_1}{\rho_0^3}\nabla\rho_0(\nabla\rho_0)^2 \end{array} \right\}. \tag{14}$$

Now, where $\vec{U}_1 = (u_{x1}, u_{y1}, u_{z1})$, $\vec{g} = (0, 0, g)$, $\vec{Q}_1 = (Q_{x1}, Q_{y1}, Q_{z1})$, $\nabla = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z})$ and the fluid is arranged in horizontal strata, then ρ_0 is a function of the vertical coordinate (z) only ($\rho_0 = \rho_0(z)$). So, the system of equations (10)-(13) can be put as:

$$\rho_0 \omega u_{x1} = -\frac{\partial p_1}{\partial x} + Q_{x1}, \tag{15}$$

$$\rho_0 \omega u_{y1} = -\frac{\partial p_1}{\partial y} + Q_{y1}, \tag{16}$$

$$\rho_0 \omega u_{z1} = -\frac{\partial p_1}{\partial z} + \rho_1 g + Q_{z1}\,, \tag{17}$$

$$\frac{\partial u_{x1}}{\partial x} + \frac{\partial u_{y1}}{\partial y} + \frac{\partial u_{z1}}{\partial z} = 0\,, \tag{18}$$

$$\omega \rho_1 + \left(\frac{d\rho_0(z)}{dz}\right) u_{z1} = 0\,, \tag{19}$$

$$\omega \eta_1 = u_{z1}\big|_{z=0} \tag{20}$$

Here Q_{x1}, Q_{y1} and Q_{z1} are given, respectively

$$Q_{x1} = \frac{\hbar^2}{2 m_e m_i} \frac{\partial}{\partial x} \left\{ \begin{array}{l} \dfrac{1}{2}\dfrac{d^2\rho_1}{dz^2} - \dfrac{1}{2\rho_0}\dfrac{d\rho_0}{dz}\dfrac{d\rho_1}{dz} + \\ + \left\{ \dfrac{1}{2}\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}\right) - \dfrac{1}{2\rho_0}\dfrac{d^2\rho_0}{dz^2} + \dfrac{1}{2\rho_0^2}\left(\dfrac{d\rho_0}{dz}\right)^2 \right\} \rho_1 \end{array} \right\}, \tag{21}$$

$$Q_{y1} = \frac{\hbar^2}{2 m_e m_i} \frac{\partial}{\partial y} \left\{ \begin{array}{l} \dfrac{1}{2}\dfrac{d^2\rho_1}{dz^2} - \dfrac{1}{2\rho_0}\dfrac{d\rho_0}{dz}\dfrac{d\rho_1}{dz} + \\ + \left\{ \dfrac{1}{2}\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}\right) - \dfrac{1}{2\rho_0}\dfrac{d^2\rho_0}{dz^2} + \dfrac{1}{2\rho_0^2}\left(\dfrac{d\rho_0}{dz}\right)^2 \right\} \rho_1 \end{array} \right\}, \tag{22}$$

$$Q_{z1} = \frac{\hbar^2}{2 m_e m_i} \left\{ \begin{array}{l} \dfrac{1}{2}\dfrac{d^3\rho_1}{dz^3} - \dfrac{1}{\rho_0}\dfrac{d\rho_0}{dz}\dfrac{d^2\rho_1}{dz^2} + \\ \left\{ \dfrac{1}{2}\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}\right) - \dfrac{1}{\rho_0}\dfrac{d^2\rho_0}{dz^2} + \dfrac{3}{2\rho_0^2}\left(\dfrac{d\rho_0}{dz}\right)^2 \right\} \dfrac{d\rho_1}{dz} + \\ \left\{ -\dfrac{1}{2\rho_0}\dfrac{d\rho_0}{dz}\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}\right) + \dfrac{1}{2\rho_0^2}\dfrac{d\rho_0}{dz}\dfrac{d^2\rho_0}{dz^2} - \dfrac{1}{2\rho_0^3}\left(\dfrac{d\rho_0}{dz}\right)^3 \right\} \rho_1 \end{array} \right\}. \tag{23}$$

Now, we put

$$X_1(x,y,z)=X_1(z)\cos\psi \text{ for } X_1 \text{ represents } u_{z1}, P_1 \text{ and } \rho_1, \tag{24}$$

$$X_1(x,y,z)=X_1(z)\sin\psi \text{ for } X_1 \text{ represents } u_{x1} \text{ and } u_{y1}, \tag{25}$$

where $\psi=k_x x+k_y y$, k_x and k_y are horizontal components of the wave-number vector $\vec{k}$ such that $k^2=k_x^2+k_y^2$..

Using the expressions (24) and (25) in the system of Eqs. (15)-(20), we have

$$\rho_0\,\omega u_{x1}(z)\sin(\psi)=k_x p_1(z)\sin(\psi)+\overline{Q}_{x1}, \tag{26}$$

$$\rho_0\,\omega u_{y1}(z)\sin(\psi)=k_y p_1(z)\sin(\psi)+\overline{Q}_{y1}, \tag{27}$$

$$\rho_0\,\omega u_{z1}(z)\cos(\psi)=-\frac{dp_1(z)}{dz}\cos(\psi)+\rho_1(z)g\cos(\psi)+\overline{Q}_{z1}, \tag{28}$$

$$k_x u_{x1}(z)+k_y u_{y1}(z)+\frac{du_{z1}(z)}{dz}=0, \tag{29}$$

$$\rho_1(z)=-\frac{1}{\omega}u_{z1}(z)\,\frac{d\rho_0}{dz}, \tag{30}$$

$$\omega\eta_1=u_{z1}\big|_{z=0}, \tag{31}$$

where $\overline{Q}_{x1}$, $\overline{Q}_{y1}$ and $\overline{Q}_{z1}$ are given, respectively,

$$\overline{Q}_{x1}=\frac{\hbar^2 k_x}{2m_e m_i}\left\{\begin{array}{l}\frac{1}{2}\frac{d\rho_0}{dz}\frac{d^2u_{z1}}{dz^2}+\left(\frac{d^2\rho_0}{dz^2}-\frac{1}{2\rho_0}\left(\frac{d^2\rho_0}{dz^2}\right)^2\right)\frac{du_{z1}}{dz}+\\ \left(\frac{1}{2}\frac{d^3\rho_0}{dz^3}-\frac{1}{\rho_0}\frac{d\rho_0}{dz}\frac{d^2\rho_0}{dz^2}-\frac{k^2}{2}\frac{d\rho_0}{dz}+\frac{1}{2\rho_0^2}\left(\frac{d\rho_0}{dz}\right)^3\right)u_{z1}\end{array}\right\}\sin\psi, \tag{32}$$

$$\overline{Q}_{y1}=\frac{k_y}{k_x}\overline{Q}_{x1}, \tag{33}$$

$$\overline{Q}_{z1}=\frac{\hbar^2}{2m_em_i}\left[\begin{array}{l}\frac{1}{2}\frac{d\rho_0}{dz}\frac{d^3u_{z1}}{dz^3}+\left(\begin{array}{l}\frac{3}{2}\frac{d^2\rho_0}{dz^2}-\\ \frac{1}{\rho_0}\left(\frac{d^2\rho_0}{dz^2}\right)^2\end{array}\right)\frac{d^2u_{z1}}{dz^2}+\left(\begin{array}{l}\frac{3}{2}\frac{d^3\rho_0}{dz^3}-\frac{3}{\rho_0}\frac{d\rho_0}{dz}\frac{d^2\rho_0}{dz^2}\\ -\frac{k^2}{2}\frac{d\rho_0}{dz}+\frac{3}{2\rho_0^2}\left(\frac{d\rho_0}{dz}\right)^3\end{array}\right)\frac{du_{z1}}{dz}\\ +\left(\begin{array}{l}\frac{1}{2}\frac{d^4\rho_0}{dz^4}-\frac{1}{\rho_0}\frac{d\rho_0}{dz}\frac{d^3\rho_0}{dz^3}-\frac{k^2}{2}\frac{d^2\rho_0}{dz^2}-\frac{1}{\rho_0}\left(\frac{d^2\rho_0}{dz^2}\right)^2+\\ \frac{5}{2\rho_0^2}\left(\frac{d\rho_0}{dz}\right)^2\frac{d^2\rho_0}{dz^2}+\frac{k^2}{2\rho_0}\left(\frac{d\rho_0}{dz}\right)^2-\frac{1}{\rho_0^3}\left(\frac{d\rho_0}{dz}\right)^4\end{array}\right)u_{z1}\end{array}\right]. \quad (34)$$

Now, multiplying Eq. (26) by k_x and Eq. (27) by k_y and adding the products with helping equation (30) we get

$$p_1(z)=\frac{\rho_0\,\omega}{k^2}\left\{-\frac{du_{z1}(z)}{dz}\right\}+\frac{\hbar^2}{4m_em_i}\left\{\begin{array}{l}\frac{d^2\rho_1}{dz^2}-\frac{1}{\rho_0}\frac{d\rho_0}{dz}\frac{d\rho_1}{dz}+\\ \left[-k^2-\frac{1}{\rho_0}\frac{d^2\rho_0}{dz^2}+\frac{1}{\rho_0^2}\left(\frac{d\rho_0}{dz}\right)^2\right]\rho_1\end{array}\right\} \quad (35)$$

Eliminating p_1 between equations (28) and (35) with helping the system of equations (30)-(34) we get a differential equation in u_{z1}

$$\left[\rho_0\omega^2+k^2A\right]\frac{d^2u_{z1}}{dz^2}+\left[\omega^2\left(\frac{d\rho_0}{dz}\right)+k^2B\right]\frac{du_{z1}}{dz}-k^2\left[\rho_0\omega^2+(C+g)\frac{d\rho_0}{dz}\right]u_{z1}=0, \quad (36)$$

where A, B and C are given, respectively,

$$\begin{aligned}A&=-\frac{\hbar^2}{4m_em_i}\frac{1}{\rho_0}\left(\frac{d\rho_0}{dz}\right)^2,\\ B&=\frac{\hbar^2}{4m_em_i}\frac{1}{\rho_0^2}\left(\frac{d\rho_0}{dz}\right)\left\{\left(\frac{d\rho_0}{dz}\right)^2-2\rho_0\frac{d^2\rho_0}{dz^2}\right\},\\ C&=\frac{\hbar^2}{4m_em_i}\frac{k^2}{\rho_0}\left(\frac{d\rho_0}{dz}\right),\end{aligned} \quad (37)$$

For the vacuum case, the linear equations may be written as follows:

$$\nabla^2 \varphi_1 = 0 \,, \quad \vec{\nabla} \times \vec{\nabla} \varphi_1 = 0 \,. \tag{38}$$

Now, if we put $\varphi_1(x, y, z) = \varphi_1(z) \sin\psi$, then Eq. (38) takes the form

$$\left[\frac{d^2}{dz^2} - k^2 \right] \varphi_1(z) = 0 \tag{39}$$

The linear case of Eqs. (7) and (8), respectively, takes the form:

$$P_1 + \eta_1 \frac{dP(0)}{dz} = \frac{B_0 \, B_{x1}}{\mu} \,, \quad \frac{dP(0)}{dz} = \rho_0(z) \, g \,, \tag{40}$$

$$\vec{n}_0 \cdot \vec{B}_1 + \vec{n}_1 \cdot \vec{B}_0 = 0 \,, \qquad \vec{n}_0 = \vec{e}_z \text{ and } \vec{n}_1 = -\frac{\partial \eta_1}{\partial x} \vec{e}_x - \frac{\partial \eta_1}{\partial y} \vec{e}_y \tag{41}$$

Now, we will introduce the solutions for the two regions, where plasma layer supported by vacuum layer that has been permeated by a uniform magnetic field in the x – direction $\left(\vec{B}_0 = B_0 \, \vec{e}_x\right)$. Plasma region (Region (I)) through the range $Z < 0$, while the vacuum region (Region (II)) through the range $0 < Z$. Region (I) $(-\infty < Z < 0)$:

Here, we consider the density distribution is given by $\rho_0(z) = \rho_0(0) \exp(z / L_D)$, where $\rho_0(0)$ and L_D are constants, so Eq. (36) becomes

$$\left(\omega^2 + \omega_q^2\right) \frac{d^2 u_{z1}(z)}{dz^2} + \frac{1}{L_D} \left(\omega^2 + \omega_q^2\right) \frac{du_{z1}(z)}{dz} - k^2 \left\{ \left(\omega^2 + \omega_q^2\right) + \frac{g}{L_D} \right\} u_{z1}(z) = 0, \tag{42}$$

$\omega_q^2 = \dfrac{\hbar^2 k^2}{4 L_D^2 m_e m_i}$, $\hbar$ is the Plank's constant, m_i the ion mass and m_e is the electron mass.

The solution of last equation in this region will be as following:

$$u_{z1}(z) = a \, \exp(q_1 z) \,, q_1 = -\frac{1}{2} \left\{ \frac{1}{L_D} - \sqrt{ \frac{1}{L_D^2} + 4k^2 \left(1 + \frac{g}{L_D \left(\omega^2 + \omega_q^2\right)} \right) } \right\} \,, \tag{43}$$

if we select $a = 1$, we will find that:

$$
\begin{aligned}
&u_{x1}(x,y,z)=-\frac{k_x}{k^2}q_1\exp(q_1z)\sin\psi,\\
&u_{y1}(x,y,z)=-\frac{k_y}{k_x}u_{x1}(x,y,z),\\
&u_{z1}(x,y,z)=\exp(q_1z)\cos\psi,\\
&\rho_1(x,y,z)=-\frac{\rho_0(z)}{\omega L_D}\exp(q_1z)\cos\psi,\\
&P_1(x,y,z)=-\rho_0(z)\left\{\frac{\omega q_1}{k^2}\exp(q_1z)+\frac{\sigma_q^2}{\sigma k^2}\left(L_D(q_1^2-k^2)+q_1\right)\right\}\cos\psi,\\
&\eta_1(x,y,z)=\frac{1}{\omega}\cos\psi.
\end{aligned}
\tag{44}
$$

Region (II) $(0<Z<\infty)$:

The solution of Eq. (39) in this region is $\varphi_1(z)=b\exp(-kz)$, and then

$$
\varphi_1(x,y,z)=\left\{b\exp(-kz)\right\}\sin\psi.\text{So,}\quad B_{z1}(x,y,z)=\left\{-bk\exp(-kz)\right\}\sin\psi. \tag{45}
$$

Using the condition (41) we find $b_1=\dfrac{B_0k_x}{k\omega}$, then we find

$$
\begin{aligned}
&B_{x1}(x,y,z)=\left\{\frac{B_0k_x^2}{k\omega}\exp(-kz)\right\}\cos\psi,\\
&B_{y1}(x,y,z)=\left\{\frac{B_0k_xk_y}{k\omega}\exp(-kz)\right\}\cos\psi,\\
&B_{z1}(x,y,z)=\left\{-\frac{B_0k_x}{\omega}\exp(-kz)\right\}\sin\psi.
\end{aligned}
\tag{46}
$$

Now, using the results (44) and (46) in the condition (41), then the dispersion relation for our problem (unmagnetized quantum plasma layer has been supported by uniform magnetized vacuum layer) given by

$$
\omega^2-\frac{k^2}{q_1}\left\{g-\frac{B_0^2k_x^2}{k\mu\rho_0(0)}-\frac{\omega_q^2}{k^2}\left(L_D(q_1^2-k^2)+q_1\right)\right\}=0. \tag{47}
$$

If we ignore the quantum effect, then the dispersion relation (47) becomes

$$\omega^4 + \frac{1}{L_D}\left\{\frac{B_0^2\,k_x^2}{k\mu\rho_0(0)}\right\}\omega^2 - k^2\left\{g - \frac{B_0^2\,k_x^2}{k\mu\rho_0(0)}\right\}^2 = 0, \tag{48}$$

while, if we ignore both quantum and magnetic field effects, we have the Rayleigh-Taylor instability mode that is given by the classical expression $\omega=(k\,g)^{\frac{1}{2}}$ see Eq. (1) ref. [12].

Now, we define the following dimensionless quantities

$$\omega^{*^2} = \frac{\omega^2}{\omega_{pe}^2},\;\; \omega_q^{*^2} = \frac{\hbar^2}{4L_D^4 m_e m_i \omega_{pe}^2},\;\; g^* = \frac{g}{\omega_{pe}^2\,L_D},\;\; k^{*2} = k^2 L_D^2,\;\; \omega_m^{*^2} = \frac{B_0^2\,k_x^2}{\mu\rho_0(0)\,\omega_{pe}^2}.$$

Then the dispersion relation (48) becomes

$$\omega^{*^2} - \frac{k^{*^2}}{q_1^*}\left\{g^* - \frac{\omega_m^{*^2}}{k^*} - \frac{\omega_q^{*^2}}{k^{*^2}}\left(q_1^{*^2} - k^{*^2} + q_1^*\right)\right\} = 0, \tag{49}$$

$$q_1^* = -\frac{1}{2}\left\{1 - \sqrt{1 + 4k^{*^2}\left(1 + \frac{g^*}{\omega^{*^2} + \omega_q^{*^2}}\right)}\right\}. \tag{50}$$

From Eq. (49) it is not feasible to obtain analytically the value of ω^* for the sets of parameters (ω_q^*, ω_m^*). So, the dimensionless dispersion relation (49) has been solved numerically for different values of the physical parameters involved. Numerical calculations are presented in Fig. 1. There the square normalized growth rate ω^{*^2} is plotted against the square normalized wave number k^{*^2}. Fig. 1 shows the role of quantum term $\omega_q^*\,(=1,\,2)$ and magnetic field $\omega_m^*(=1,2)$ for a quantum plasma-vacuum model, where the magnitudes of square normalized growth rate is less than its counterpart in the case of quantum term only or magnetic field only.

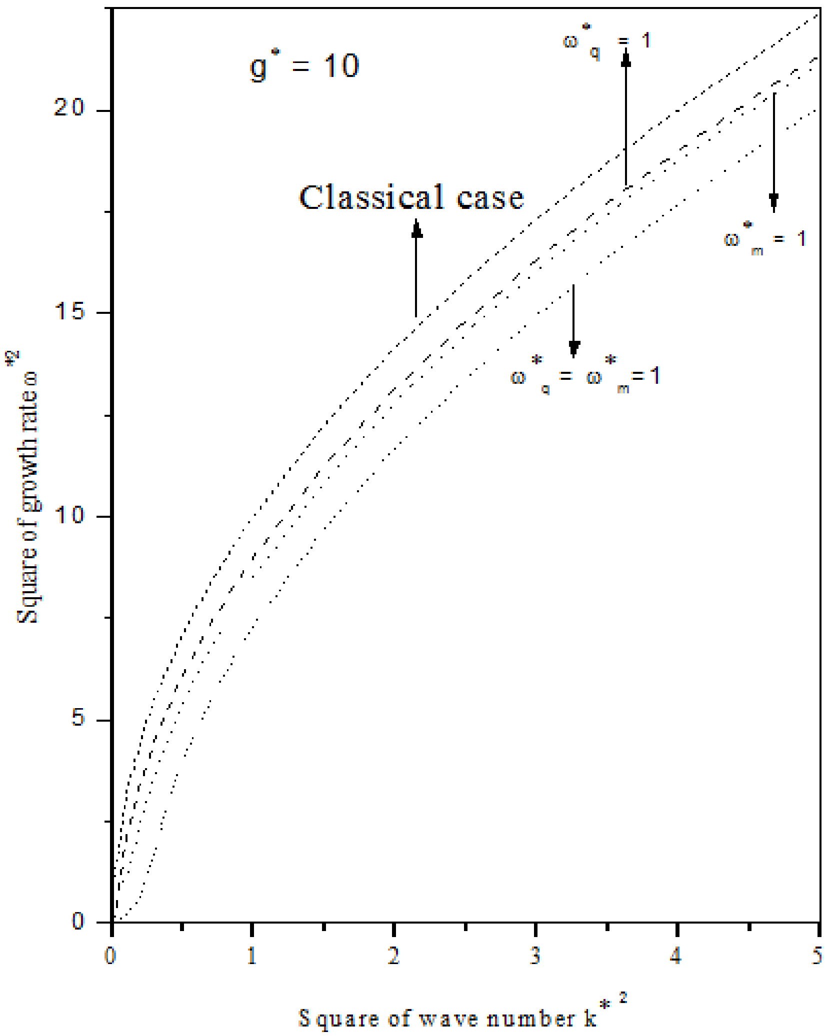

Fig. 1. The square normalized growth rate ω^{*^2} against the square normalized wave number k^{*^2} with the parameters $\omega_q^* = \omega_m^* = 1$, $h^* = 1$ and $g^* = 10$

3. CONCLUSION

Finally, we have presented the analytical results of the Rayleigh-Taylor instability of quantum plasma with vacuum interface, the dispersion relation is derived as a function of the physical parameters of the system considered in Eq. (49). The numerical calculations are shown that the system was more stability in the presence of both quantum term and horizontal magnetic field. This stabilizing, that happens in the presence of both quantum term and horizontal magnetic field. This discrepancy highlights a stabilizing role due to the presence of quantum term and horizontal magnetic field on Rayleigh-Taylor instability problem (quantum plasma-vacuum), increasing the dissipation of any disturbance, thus providing an increased stability.

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COMPETING INTERESTS

Author has declared that no competing interests exist.

REFERENCES

1. Klimontovich Y, Silin VP. Zhurnal Eksperimental'noii TeoreticheskoiFiziki (Zh. Eksp. Teor. Fiz.). 1952;23:151-154.
2. Gell-Mann M, Bruckner KA. Correlation energy of an electron gas at high density. Phys. Rev. 1957;106(2):364-368.
3. Englert F, Brout R. Dielectric Formulation of Quantum Statistics of Interacting Particles. Phys. Rev. 1960;120(4):1085-1092.
4. Ehrenreich H, Cohen MH. Self-Consistent Field Approach to the Many-Electron Problem. Phys. Rev. 1959;115(4):786-790.
5. Balescu R. Approach to Equilibrium of a Quantum Plasma. Phys. Fluids. 1960;4(1):94-99.
6. Guernsey RL. Kinetic Theory of Quantum Plasmas. Phys. Rev.1962;127(5):1446-1452.
7. Pines D. Classical and quantum plasmas. Journal of Nuclear Energy. Part C, Plasma Physics, Accelerators, Thermonuclear Research. 1961;2:5-17.
8. Zyryanov PS, Okulov VI, Silin VP. Quantum Spin Waves. JETP Letters. 1968;8(9):300-301.
9. Zyryanov PS, Okulov VI, Silin VP. Quantum Electronic Spin Acoustic Waves. JETP Letters. 1969;9(6):220-222.
10. Chabrier G, Douchin F, Potekhin AY. Dense astrophysical plasmas. J. Phys.: Condens. Matter. 2002;14:9133-9139.
11. Marklund M, Shukla PK. Nonlinear collective effects in photon-photon and photon plasma interactions, Rev. Mod. Phys. 2006;78:591-640.
12. Goedbloed J.P. Plasma-vacuum interface problems in magnetohydro-dynamics. Physica. 1984;12(D):107-132.
13. Velikovich AL. Rayleigh–Taylor instability of a plasma–vacuum boundary in the limit of a large Larmor radius. Physics of Plasmas. 1991;3(2):492-494.
14. GrattojnT, Gratton FT, Gonzalez AG. Convective instability of internal modes in accelerated compressible plasmas. Plasma Physics and Controlled Fusion. 1988;30(4):435.
15. Alejandro G, Gratton J. Compressibility effects on the gravitational instability of a plasma-vacuum interface. Plasma Physics and Controlled Fusion. 1990;32(1):3-20.
16. Stenflo L. Theory of nonlinear plasma surface waves. Phys. Scripta. 1996;T63:59-62.
17. Stenflo L, Yu MY. Oscillons at a plasma surface. Phys. Plasmas. 2003;10(3):912-913.
18. Lee HJ. Comment on "Kinetic theory of surface waves in plasma jets" [Phys. Plasmas 9, 701 (2002)]. Phys. Plasmas. 2005;12:094701.
19. Lazar M, Shukla PK, Smolyakov A. Surface waves on a quantum plasma half-space. Physics of plasma. 2007;14:124501.
20. Bahaa Mohamed F. Quantum effects on the propagation of surface waves in magnetized plasma. Phys. Scr. 2010;82:065502.

21. Mohamed B. F, Abdel Aziz M. International Journal of Plasma Science and Engineering. Propagation of TE-Surface Waves on Semi-Bounded Quantum Plasma. 2010; Article ID 693049.
22. Misra AP, Ghosg NK, Shukla PK. Surface waves in magnetized Quantum electron-positron plasmas. J. Plasma Phys. 2010;76(1):87–99.
23. Misra AP. Electromagnetic surface modes in a magnetized quantum electron-hole plasma. Physical Review E. 2011;83:057401.
24. Callebaut DK. Lineaire en niet-lineaire perturbaties in hydromagneto-en Gravito dynamica, Instituut voor Nukleaire Wetenschappen, Rijksuniversiteit te Gent, Simon Stevin; 1971.
25. Hoshoudy GA. Quantum effects on the Rayleigh–Taylor instability of stratified fluid/plasma through porous media. Physics letters A. 2009;373:2560-2567.
26. Hoshoudy GA. Quantum Effects on the Rayleigh-Taylor Instability of Viscoelastic Plasma Model through a Porous Medium. Journal of Modern Physics. 2011;2:1146-1155.

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