
*Journal of Scientific Research & Reports*

*19(3): 1-9, 2018; Article no.JSRR.40909*
*ISSN: 2320-0227*

# A Probabilistic Application of Generalized Linear Model in Discrete-Time Stochastic Series

**Imoh Udo Moffat[1*] and Emmanuel Alphonsus Akpan[2]**

[1]*Department of Mathematics and Statistics, University of Uyo, Nigeria.*
[2]*Department of Mathematical Science, Abubakar Tafawa Balewa University, Nigeria.*

***Authors' contributions***

*This work was carried out in collaboration between both authors. Author IUM designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Author EAA managed the analyses of the study and the literature searches. Both authors read and approved the final manuscript.*

***Article Information***

DOI: 10.9734/JSRR/2018/40909
*Editor(s):*
(1) Martin J. Bohner, Professor, Missouri University of Science and Technology, Rolla, Missouri, USA.
*Reviewers:*
(1) Yekti Widyaningsih, University of Indonesia, Indonesia.
(2) Thomas L. Toulias, Technological Educational Institute of Athens, Greece.
Complete Peer review History: http://www.sciencedomain.org/review-history/24993

***Original Research Article***

***Received 5th February 2018***
***Accepted 12th April 2018***
***Published 5th June 2018***

## ABSTRACT

This study is aimed at identifying the problem associated with Ordinary Least Squares (OLS) in relation to the violation of assumptions of normality and constant variance. Mainly, the possible problem encountered when these assumptions are violated is the introduction of biases in the parameters of the fitted model thereby threatening the model's efficiency. In this study, the Generalized Linear Model (GLM) is applied to overcome such problems and to ensure the efficiency of the model parameters. The major reasons being that the GLM does not require transformation and assumptions of classical regression. Instead, it employs a probabilistic approach in transforming the expected value of the dependent variable. The data used were obtained from the Central Bank of Nigeria Statistical Bulletin from 1981 to 2016, with each series consisting of 36 observations. The Gross Domestic Product (N' Billion) was considered as the dependent variable ($\mathrm{Y_t}$) while Money Supply($\mathrm{X_{1t}}$), and Credit to Private Sector($\mathrm{X_{2t}}$)were considered as the independent variables (N' Billion). From the analysis, the results of the fitted regression model showed no significant relationship between the variables. The diagnosis on the residual series (using skewness, kurtosis, Jacque-Bera test and Breusch-Pagan-Godfrey test) provided

**Corresponding author: E-mail: moffitto2011@gmail.com;*

sufficient evidence that both validity and efficiency of the model parameters are threatened. However, the results of the GLM procedure provided the much needed significance, validity, and efficiency of the model parameters. Further findings from GLM procedure revealed that the standard errors of the parameters of OLS were biased having been far larger in values than those of the GLM. Hence, for studies involving the regression of a discrete-time stochastic series such as GDP on Money Supply and Credit to Private Sector, the GLM is analytically tractable than the OLS.

*Keywords: Homoscedasticity; linear model; normal distribution; ordinary least squares; poisson regression.*

## 1. INTRODUCTION

Ordinary Least Squares (OLS) is an estimation method in classical regression analysis which seeks to use the criterion that the solution of a simple linear model must give the smallest possible sum of squared deviations of the observed dependent variable, say $Y_t$ from the estimates of their true means provided by the solution (see [1,2]). Intriguingly, the OLS method has become the most commonly used methods in regression analysis due to the attractive properties associated with its estimators based on certain assumptions about the way the data were generated. The ordinary estimator can easily be calculated since they are expressed mainly in terms of observable quantities; this is one property that popularized the application of ordinary least squares method. Other properties include:

a. In any given sample, each estimator will produce only a single point value of the relevant population parameter (that is, OLS estimators are point estimators).
b. Once the least squares estimates are obtained from the sample data, the sample regression line is obtained and such regression line passes through the sample means of the dependent and independent variables [2].

Also, the basic underlying assumptions of least squares are as follows:

1. That the errors of the regression model must be normally distributed with zero mean and constant variance
2. That the variance of the errors must be homogeneous
3. That the error term must be uncorrelated.

Despite the huge success and popularity of ordinary least squares method, with the violation of these basic assumptions, the least squares estimators become inefficient, standard errors of the parameters would not be correct and the t-test statistic would be invalid [3,1,2,4].

However, one way of handling the problem of the OLS, is to undertake the transformations of dependent or independent or both dependent and independent variables. According to [1], the basic reason for transformations of the dependent variables is to remedy for non-normality and heterogeneity in the variables and for simplification of the relationship between the dependent and the independent variables. Meanwhile, [5] pointed out that the use of transformations suggested by data still leads to problem of interpretation. Using transformed variables for regression analysis equally failed to provide inference on the targeted population.

To avoid the problem of OLS along with that of transformations, this study seeks to adopt a Generalized Linear Model (GLM) approach which generalizes linear regression by allowing the linear model to be related to the response variable through a link function and by allowing the magnitude of the variance of each measurement to be a function of its predicted value. The difference between the GLM and OLS is: by particular model, the response distribution of one of the exponential family of distributions (which includes Normal, Poisson, Gamma, Binomial, Inverse Gaussian) and the link function (Identity, Logarithmic, Square root, Logistic, Power) which relates the mean of the response to a scale on which the model effects are combined additively [6].

The motivation for this study is based on the fact that previous studies involving Money Supply, Credit to Private Sector and economic growth in Nigeria for instance, [7,8,9,10,11,12,13,14] and [15] have applied OLS method but failed to account for the possible violation of assumptions of normality and homoscedasticity. Particularly, a reserved attention is given to the work of [16] who studied and modeled the autocorrelated errors of the regression of GDP on Money Supply and Credit to Private Sector using generalized least squares method on a yearly data set ranging from 1981 to 2014. Their findings revealed that generalized least squares method outperformed the ordinary least squares method and in addition captured the unexplained variance embedded in the error term by AR(2) process. They recommended a possible check on a possible violation of assumption of homoscedasticity. Hence, in this study, we seek to improve on the work of [16] by updating the data set from 1981 to 2016 and adopting the generalized linear model to overcome the problem associated with the violation of assumption of normality and homoscedasticity. The reason for adopting the generalized linear model is to complement the generalized least squares method in overcoming the weaknesses of ordinary least squares method given that the generalized linear model provides the needed solution to the violation of assumptions of normality and homoscedasticity while generalized least squares method accounts for the presence of autocorrelated errors.

## 2. MATERIALS AND METHODS

### 2.1 Regression Model

According to [1], the standard regression model is as follows:

$$Y_t = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + \cdots + \beta_k X_{kt} + \varepsilon_t, \quad (1)$$

where

$Y_t$ = dependent variable
$\beta_i$ = regression parameters, i = 1,…, k
$X_{it}$ = independent variables, i = 1,…, k
$\varepsilon_t$ = error term assumed to be i.i.d. N(0, $\sigma_t^2$)

(See also [3,17,18,19]). Thus, the dependent variable for a time series regression model with independent variables is a linear combination of independent variables measured in the same time frame as the dependent variable. Estimates of the parameters of the model in (1) can be obtained by Least Squares Estimation Method.

### 2.2 Method of Ordinary Least Squares for Simple Regression

The least squares estimation procedure uses the criterion that the solution must give the smallest possible sum of squared deviations of the observed $Y_i$ from the estimates of their true means provided by the solution. Let $\hat{\beta}_o$ and $\hat{\beta}_1$ be numerical estimates of the parameters $\beta_0$ and $\beta_1$, respectively, and let:

$$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i, \quad (2)$$

be the estimated mean of Y for each $X_i$, $i = 1, \ldots$, k.

The least squares principle chooses $\hat{\beta}_0$ and $\hat{\beta}_1$ that minimize the sum of squares of the residuals, SSE:

$$\text{SSE} = \sum_{i=1}^{k} \left(Y_i - \hat{Y}_i\right)^2 = \sum_{i=1}^{k} \varepsilon_i^2, \quad (3)$$

where $\varepsilon_i = (Y_i - \hat{Y}_i)$ is the observed residual for the $ith$ observation.

Also, we can express $\varepsilon_i$ in terms of $Y_i$, $X_i$, $\beta_0$, and $\beta_1$. Hence, we have:

$$\varepsilon_i = Y_i - \beta_0 - \beta_1 X_i, \quad (4)$$

and (4) becomes:

$$\text{SSE} = \sum_{i=1}^{k} (Y_i - \beta_0 - \beta_1 X_i)^2. \quad (5)$$

The partial derivative of SSE with respect to the regression constant, $\beta_0$, that is:

$$\frac{\partial SSE}{\partial \beta_0} = \frac{\partial}{\partial \beta_0}\left[\sum_{i=1}^{k} (Y_i - \beta_0 - \beta_1 X_i)^2\right], \quad (6)$$

with some subsequent rearrangement, the estimate of $\beta_0$ is obtained as:

$$\hat{\beta}_0 = \left[\frac{\sum_{i=1}^{k} Y_i}{k}\right] - \beta_1 \left[\frac{\sum_{i=1}^{k} X_i}{k}\right]. \quad (7)$$

The partial derivative of SSE with respect to the regression coefficient, $\beta_1$, that is:

$$\frac{\partial SSE}{\partial \beta_1} = \frac{\partial}{\partial \beta_1}\left[\sum_{i=1}^{k} (Y_i - \beta_0 - \beta_1 X_i)^2\right], \quad (8)$$

rearranging (8), we obtain the estimate of $\beta_1$ as:

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{k} Y_i X_i - \frac{\sum_{i=1}^{k} Y_i \sum_{i=1}^{k} X_i}{k}}{\sum_{i=1}^{k} X_i^2 - \frac{\left(\sum_{i=1}^{k} X_i\right)^2}{k}}, \quad (9)$$

See [20,1,2,17] for more details on Least Squares Estimation Method.

### 2.3 Diagnostic Test for Heteroscedaticity

Breusch-Pagan-Godfrey (BPG) test is used to test for the presence of heteroscedasticity in the residuals of a fitted regression model. The test statistic is given as:

$$\text{BPG} = \frac{1}{2}\ (\text{ESS} = \sum(\hat{\varepsilon} - \bar{\varepsilon})^2) \approx \chi_p^2\ , \quad (10)$$

and under the null, ESS is the Explained Sum of Squares, $\hat{\varepsilon}$ is the predicted value of $\varepsilon$ (residual term) and $\bar{\varepsilon}$ is the mean of $\varepsilon$ (residual term)in the regression:

$$\sigma_i^2 = \alpha_1 + \alpha_2\varepsilon_{1i} + \cdots + \alpha_p\varepsilon_{pi}, \quad (11)$$

where $\sigma_i^2$ is a function of the non-stochastic variables $\varepsilon$ 's. If $\alpha_2 = \alpha_3 = \cdots = \alpha_P = 0$ , $\sigma_i^2 = \alpha_1$, which is a constant. Therefore, to test whether $\sigma_i^2$ is homoscedastic, one can test the hypothesis that $\alpha_2 = \alpha_3 = \cdots = \alpha_P = 0$. If the calculated value of the BPG exceeds the critical $\chi^2$ value at 5% level of significance, the hypothesis of homoscedasticity can be rejected; otherwise the hypothesis is not rejected (see [21, 22,23,2]).

### 2.4 Generalized Linear Model

Generalized linear models as formulated by [24] provide a generalization of ordinary least squares expression that relates the random term (which is a linear predictor $X_i^{'}\beta$) via a link function denoted by $g(\mu_i)$ . Supposing $y_1, \dots, y_p$ denote $p$ independent observations on a response. Note that$y_{i,}\ i = 1, 2, \dots, p$ is considered as a realization of a random variable $Y_i$. For the general linear model, it is assume to follow a normal distribution with mean $\mu_i$ and variance $\sigma^2$, that is $Y_i \sim N(\mu_i, \sigma^2)$. It is also assumed that the expected value $\mu_i$ is a linear function of $k$ predictors that take the values $X_i^{'}$ = $(y_{i1}, \dots, y_{ik})$ for the $i\text{-}th$ case, so that $\mu_i = X_i^{'}\beta$, where $\beta$ is a vector of unknown parameters.

In relation to the link function, a one-to-one continuous differentiable transformation $g(\mu_i)$ is introduced. It is noteworthy that in generalized linear model, it is the expected value $\mu_i$that is being transformed not the response $y_i$. Assuming that the transformed mean follows a linear model, then:

$$\text{E}(Y_i) = \mu_i = g^{-1}(X_i^{'}\beta), \quad (12)$$

$$g(\mu_i) = X_i^{'}\beta. \quad (13)$$

Some common link functions identified are:

The Identity Link:

$$g(\mu_i) = X_i^{'}\beta, \quad (14)$$

which is used when the error follows a normal distribution. It is commonly used in a traditional regression.

The Logit Link:

$$g(\mu_i) = \log\left(\frac{\mu_i}{1-\mu_i}\right) = X_i^{'}\beta. \quad (15)$$

Equivalently,

$$\mu_i = \frac{e^{X_i^{'}\beta}}{1-e^{X_i^{'}\beta}}.$$

This is used when the error follows a binomial distribution. It is commonly applicable in Logistic regression.

The Log Link:

$$g(\mu_i) = \log(\mu_i) = X_i^{'}\beta. \quad (16)$$

Equivalently,

$$\mu_i = e^{X_i^{'}\beta}.$$

This is used when the error follows a Poisson distribution. It is commonly applicable in Poisson regression.

## 3. RESULTS AND DISCUSSION

To ascertain the nature of the relationship between the $Y_t$ and each of $X_{1t}$ and $X_{2t}$, we regress $Y_t$ on $X_{1t}$ and $X_{2t}$ . The estimated model is shown in (17):

$$Y_t = 632.8187 + 2.5834X_{1t} + 2.2884X_{2t} \quad (17)$$

s.e (1077.045) (1.6246) (1.6403)
t-value (0.5876) (1.5901) (1.3951)
*p*-value (0.5608) (0.1213) (0.1723)
[Excerpts from Table 1].

**Table 1. Output of Regression Model**

Dependent Variable: $Y_t$
Method: Least Squares
Date: 03/14/18 Time: 14:39
Sample: 1981 2016
Included observations: 36

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|---|---|---|---|---|
| C | 632.8187 | 1077.045 | 0.587551 | 0.5608 |
| $X_{1t}$ | 2.583365 | 1.624620 | 1.590135 | 0.1213 |
| $X_{2t}$ | 2.288438 | 1.640336 | 1.395104 | 0.1723 |
| R-squared | 0.976454 | Mean dependent var | | 20235.62 |
| Adjusted R-squared | 0.975027 | S.D. dependent var | | 31208.35 |
| S.E. of regression | 4931.800 | Akaike info criterion | | 19.92445 |
| Sum squared resid | 8.03E+08 | Schwarz criterion | | 20.05641 |
| Log likelihood | -355.6401 | Hannan-Quinn criter. | | 19.97051 |
| F-statistic | 684.2591 | Durbin-Watson stat | | 1.166935 |
| Prob(F-statistic) | 0.000000 | | | |

From (17) it is observed that the constant term (which is the expected value of $Y_t$ when $X_{1t}$ and $X_{2t}$ are zero) is not significant given that its corresponding *p*-value is greater than 5% level of significance. Similarly, there appears to be no significant relationship between the dependent variable, $Y_t$ and the independent variables, $X_{1t}$ and $X_{2t}$ given that the *p*-values corresponding to $X_{1t}$ and $X_{2t}$ are greater than 5% level of significance. Having failed to establish a significant relationship between the variables and for the purpose of argument, we assessed the residuals of the model in order to ascertain the nature of the relationship and improve upon the model. The graph of the residual series in Fig. 1 shows that the relationship is nonlinear. The implication is that the fitted linear regression model is not suitable.

Another diagnostic check on the residuals of the fitted regression model is the normality test. From Fig. 2, it could be observed that both tails of the Q-Q plot deviates from the normal line indicating that the distribution is non-normal. Also, the distribution is skewed to the left as indicated by the coefficient of skewness, -2.8654 against zero which is meant to be the value accommodated by a normal distribution. Moreover, the coefficient of kurtosis is 12.8467 greater than 3, the value occupied by a normal distribution. The value of the Jarque-Bera test statistic is 194.6995 with corresponding p-value = 0.0000 which is less than 5% level of significance, implying the rejection of the null hypothesis of normal distribution. Hence we can further affirm that the fitted regression model is not a good fit given that it was fitted under the assumption of normal distribution.

The third diagnostic check on the fitted regression model is the heteroscedasticity test. From Table 2, the hypothesis of homoscedasticity is rejected given the Breusch-Pagan-Godfrey test value of 13.2359 whose corresponding p-value of 0.0013 is less than 5% level of significance, that is, ($p$ = 0.0000 < 0.05) meaning the assumption of constant variance is violated. The implication is that, the fitted model that was built under the assumption of constant variance is not a good fit.

**Table 2. Heteroskedasticity Test: Breusch-Pagan-Godfrey**

| | | | |
|---|---|---|---|
| F-statistic | 1.316052 | Prob. F(2,33) | 0.2819 |
| Obs*R-squared | 2.659280 | Prob. Chi-Square(2) | 0.2646 |
| Scaled explained SS | 13.23589 | Prob. Chi-Square(2) | 0.0013 |


**Fig. 1. Graph of Residual Series of the Regression Model**

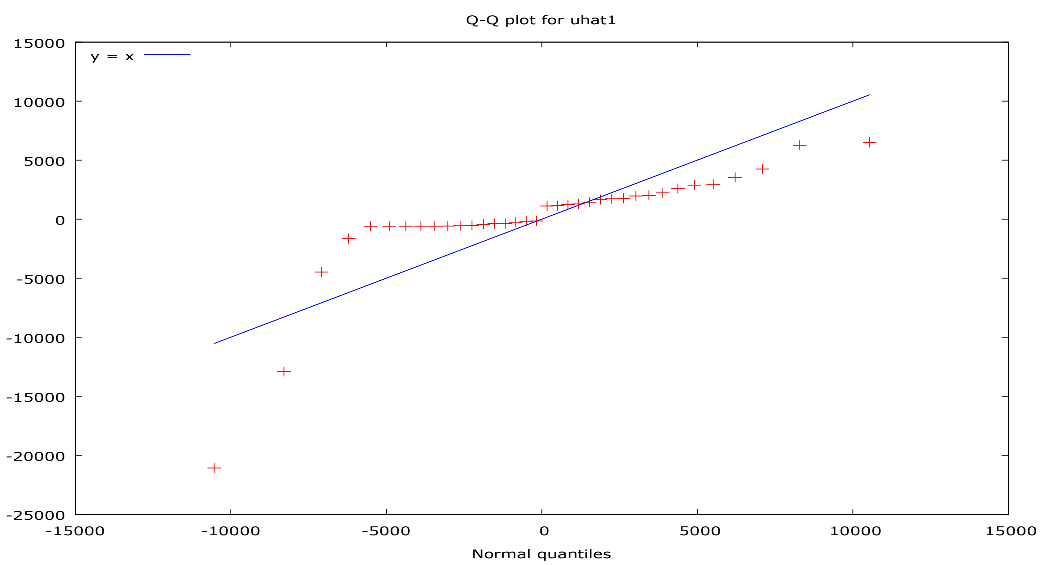


**Fig. 2. Q-Q Plot of the Residual Series**

So far, we have established that the relationship between GDP ($Y_t$) and each of Money Supply ($X_{1t}$) and Credit to Private Sector ($X_{2t}$) using OLS is not the best fit. However, to overcome the weaknesses of the OLS as related to this study, we employ the method of GLM which generalizes the ordinary linear regression to allow for response variables with error distribution models other than a normal distribution. Furthermore, a probabilistic approach which involves the log link function corresponding to a Poisson distribution is considered following the fact that GLM uses both the exponential and logarithmic functions at the same time to achieve linearity in the parameters especially where nonlinear relationship tends to exist. Hence, the fitted GLM is presented in (18):

$$Y_t = e^{(8.4621 + 0.0003X_{1t} - 0.0002X_{2t})} \quad (18)$$

| | | | |
|---|---|---|---|
| s.e | (0.0027) | $(2.22 \times 10^{-6})$ | $(2.21 \times 10^{-6})$ |
| t-value | (3138.333) | (156.0586) | (- 84.0973) |
| *p*-value | (0.0000) | (0.0000) | (0.0000) |

[Excerpts from Table 3].

**Table 3. Output of generalized linear model**

Dependent Variable: $Y_t$
Method: Generalized Linear Model (Quadratic Hill Climbing)
Date: 03/14/18 Time: 14:57
Sample: 1981 2016
Included observations: 36
Family: Poisson (quasi-likelihood)
Link: Log
Dispersion fixed at 1
Coefficient covariance computed using observed Hessian
Convergence achieved after 6 iterations

| **Variable** | **Coefficient** | **Std. Error** | **z-Statistic** | **Prob.** |
|---|---|---|---|---|
| C | 8.462138 | 0.002696 | 3138.333 | 0.0000 |
| $X_{1t}$ | 0.000346 | 2.22E-06 | 156.0586 | 0.0000 |
| $X_{2t}$ | -0.000186 | 2.21E-06 | -84.09732 | 0.0000 |
| Mean dependent var | 20235.62 | S.D. dependent var | | 31208.35 |
| Sum squared resid | 7.97E+09 | Quasi-log likelihood | | 7081739. |
| Deviance | 205166.6 | Deviance statistic | | 6217.171 |
| Restr. Deviance | 1379516. | Quasi-LR statistic | | 1174349. |
| Prob(Quasi-LR stat) | 0.000000 | Pearson SSR | | 173460.2 |
| Pearson statistic | 5256.370 | Dispersion | | 1.000000 |

From the model in (18), we observed that the intercept is significant given that the p-value is less than 5% level of significance ($p$ = 0.0000< 0.05), that is, when $X_{1t}$ and $X_{2t}$ are zero, then the value of $Y_t$ will remain at $e^{8.4\ 621} = 4731.94\ (N'Billion)$. Also, the relationship between $Y_t$ and $X_{1t}$ is found to be significant since the corresponding $p$-value is less than 5% level of significance ($p$ = 0.0000 < 0.05), that is, for any unit increase in $X_{1t}$, $Y_t$ increases by $e^{0.0003} = 1.0003\ (N'Billion)$. Similarly, the relationship between $Y_t$ and $X_{2t}$ is found to be significant in that the corresponding $p$-value is less than 5% level of significance ($p$ = 0.0000 < 0.05), that is, for a unit increase in $X_{2t}$, $Y_t$ increases by $e^{-0.0002} = 0.9998\ (N'Billion)$. The diagnostic checks on the fitted GLM revealed that the distribution is marginally skewed to the right as indicated by the coefficient of skewness, 0.6325 against zero which is meant to be the value accommodated by a normal distribution. Moreover, the coefficient of kurtosis is 3.3906 approximately equal to 3, the value occupied by a normal distribution. The value of the Jarque-Bera test statistic is 2.6292 with corresponding $p$-value = 0.2686 which is greater than 5% level of significance, implying no rejection of the null hypothesis of normal distribution. Also, evidence from Q-statistic for the squared residuals of the fitted GLM showed that assumption of homoscedasticity is not violated given that the Q-statistic value for the first 16 lags is 18.340 with its corresponding $p$-value, 0.304 which is greater than 5% level of significance. Hence, we can affirm that the fitted GLM is a good fitted model, assuming that normality and homoscedasticity are not violated. However, our main interest is in comparing the ordinary least squares with the generalized linear model, regarding the efficiency of the model parameters when the violation of assumptions of normality and homoscedasticity (constant variance) are taken into consideration. As it is evident in Table 4, the intercept of the GLM appears to be more efficient than that of the OLS, given that the standard error of the intercept of GLM is far smaller than that of the OLS model. Also, the $p$-value pertaining to the intercept of GLM is significant, that is, ($p$ = 0.0000 $< 0.05$) while the $p$-value of the OLS is not significant, that is, ($p$ = 0.5608$> 0.05$). Similarly, the coefficient, $\beta_1$ in GLM appeared more efficient with a standard error smaller than that of the OLS. The coefficient, $\beta_2$ in GLM equally appeared more efficient with a smaller standard error than that of the OLS.

**Table 4. Ordinary least square vs generalized linear model**

| Model | Ordinary Least Square (OLS) | | | Generalized Linear Model (GLM) | | |
|---|---|---|---|---|---|---|
| | $\boldsymbol{\beta_0}$ | $\boldsymbol{\beta_1}$ | $\boldsymbol{\beta_2}$ | $\boldsymbol{\beta_0}$ | $\boldsymbol{\beta_1}$ | $\boldsymbol{\beta_2}$ |
| Parameter | 632.8187 | 2.5834 | 2.2884 | 8.4621 | 0.0003 | -0.0002 |
| Standard Error (s.e) | 1077.045 | 1.6246 | 1.6403 | 0.0027 | $2.22 \times 10^{-6}$ | $2.21 \times 10^{-6}$ |
| t-value | 0.5876 | 1.5901 | 1.3951 | 3138.333 | 156.0586 | - 84.0973 |
| *p*-value | 0.5608 | 0.1213 | 0.1723 | 0.0000 | 0.0000 | 0.0000 |

## 4. CONCLUSION

This study in particular fitted a regression model of GDP ($Y_t$) on the Money Supply ($X_{1t}$) and Credit to Private Sector ($X_{2t}$) aiming at identifying problems of OLS in relation to the violation of the assumptions of normality and constant variance. The provision of the needed remedy to these problems is achieved through the GLM procedure by ensuring efficiency in the model parameters. From our findings, the fitted regression model showed no significant relationship between the dependent and independent variables with the parameters not adequate at 5% level of significance. Moreover, diagnostic checks on the residual series of the fitted regression model; Q-Q plot, skewness, kurtosis and Jarque-Bera test all revealed that the assumption of normality is violated, as well as the Breusch-Pagan-Godfrey test which showed that the assumption of homoscedasticity is also violated. Possibly, this could be the reason for the inadequacy of the parameters. However, with the validity of the fitted regression model threatened, as indicated by the diagnostic checks, the results of the GLM procedure showed that all the parameters are adequate at 5% level of significance. Also, the diagnostic checks based on skewness, kurtosis, Jarque-Bera test, and Q-statistic revealed that the assumptions of normality and homoscedasticity are not violated. The implication is that, when the OLS is applied and the assumptions of normality and homoscedasticity are violated, then biases are being introduced into the model parameters. Therefore, the GLM provides the needed solution by generalizing the OLS method. Hence, for studies involving the regression of discrete-time stochastic series, the GLM is still more tractable than the OLS regression. For a further research consideration, the effect of interaction between money supply and credit to the private sector on GDP can be evaluated.

## COMPETING INTERESTS

Authors have declared that no competing interests exist.

## REFERENCES

1. Rawlings JO, Pantula SG, Dickey DA. Applied regression analysis: A Research Tool. 2nd ed., Springer; 1998.
2. Gujarati DN. Basic econometrics. 4th ed., McGraw-Hill; 2004.
3. Pankratz A. Forecasting with dynamic regressions models. 3rd ed., New York, John Wiley and Sons; 1991.
4. Akpan EA, Moffat IU, Ekpo NB. Modeling regression with time series errors of gross domestic product on government expenditure. International Journal of Innovation and Applied Studies. 2016; 18(4):990-996.
5. Chhikara RS, Folks JL. The inverse gaussian distribution: Theory, methodology, and application. New York, Marcel Dekker, Inc.; 1989.
6. Moran JL, Solomon PJ, Peisach AS, Martin J. New model for old questions: Generalized linear models for cost prediction. Journal of Evaluation in Clinical Practice. 2006;1-9.
7. Ogunmuyiwa MS, Ekone AF. Money supply – Economic growth nexus in Nigeria. Journal of Social Sciences. 2010; 22(3):199–204.
8. Bakare AS. An empirical study of the determinants of money supply growth and its effects on inflation rate in Nigeria. Journal of Research in International Business and Management. 2011;1(15): 124-129.
9. Babatunde MA, Shuaibu MI. Money supply, inflation and economic growth in Nigeria. Asian-African Journal of Economics and Econometrics. 2011; 11(1):147-163.
10. Chinaemerem OC, Chigbiu EE. An evaluation of financial development and economic growth of Nigeria: A causality test. Kuwait Chapter of Arabian Journal of Business and Management Review. 2012; 1(10):22-44.
11. Inam U. Money supply and economic growth in Nigeria: An econometric

analysis. Journal of Economics and Sustainable Development. 2014;5(12): 149-155.
12. Usman OA, Adejare AT. Money supply, foreign exchange regimes and economic growth in Nigeria. Research Journal of Finance and Accountin. 2014;5(8):121-129.
13. Ujiju LE, Etale LM. Macroeconomic analysis of the relationship between monetary policy instruments and inflation in Nigeria. International Journal of Business and Management Review. 2016; 4(6):31-39.
14. Nwoko NM, Ihemeje JC, Anumadu E. The impact of monetary policy on the economic growth of Nigeria. African Research Review. 2016;10(3):192-206.
15. Inam US, Ime BS. Monetary policy and economic growth in Nigeria: Evidence from Nigeria, Advances in Social Sciences Research Journal. 2017;4(6):41-59.
16. Akpan EA, Moffat IU. Modeling the autocorrelated errors in time series regression: A generalized least squares approach. Journal of Advances in Mathematics and Computer Science. 2018;26(4):1-15.
17. Dalgaard P. Introductory statistics with R. 2nd ed., Springer. 2008;228.
18. Moffat IU, Akpan EA. Modeling and forecasting trend function of a discrete-time stochastic process. American Journal of Scientific and Industrial Research. 2014; 5(6):195-202.
19. Akpan EA, Moffat IU. Dynamic time series regression: A panacea for spurious correlations. International Journal of Scientific and Research Publications. 2016;6(10):337-342.
20. Drasper NR, Smith H. Applied Regression Analysis. 3rd ed., New York, John Wiley and Sons; 1998.
21. Godfrey LG. Testing for Higher order Serial Correlation in Regression Equations when the Regressors include Lagged Dependent Variables. Econometrica. 1978;46:1303-1310.
22. Breusch TS. Testing for autocorrelation in dynamic linear models. Australian Economic Papers. 1978;17:334–355.
23. Breusch TS, Pagan AR. The lagrange multiplier test and its applications to model specification in econometrics. Review of Economic Studies. 1980;47: 239–253.
24. Nelder JA, Wedderburn RWM. Generalized linear models. Journal of the Royal Statistical Society, Series A. 1972; 135:370-384.



*Peer-review history:*
*The peer review history for this paper can be accessed here:*
*http://www.sciencedomain.org/review-history/24993*