Modern Mechanical Engineering, 2017, 7, 35-56
http://www.scirp.org/journal/mme
ISSN Online: 2164-0181
ISSN Print: 2164-0165

Optimisation of Effective Design Parameters for an Automotive Transmission Gearbox to Reduce Tooth Bending Stress

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How to cite this paper: Bozca, M. (2017) Optimisation of Effective Design Parameters for an Automotive Transmission Gearbox to Reduce Tooth Bending Stress. *Modern Mechanical Engineering*, **7**, 35-56.
https://doi.org/10.4236/mme.2017.72004

Received: February 15, 2017
Accepted: May 7, 2017
Published: May 10, 2017

Abstract

Optimisation of effective design parameters to reduce tooth bending stress for an automotive transmission gearbox is presented. A systematic investigation of effective design parameters for optimum design of a five-speed gearbox is studied. For this aim contact ratio effect on tooth bending stress by the changing of contact ratio with respect to pressure angle is analysed. Additionally, profile modification effects on tooth bending stress are presented. During the optimisation, the tooth bending stress is considered as the objective function, and all the geometric design parameters such as module, teeth number etc. are optimised under two different constraints, including tooth contact stress and constant gear centre distance. It can be concluded that higher the contact ratio results in a reduced tooth bending stress, while higher the pressure angle caused an increase in tooth bending stress and contact stress, since decreases in the contact ratio. In addition, application of positive profile modification on tooth reduces tooth bending stress. All of the obtained optimum solutions satisfy all constraints.

Keywords

Optimisation, Gears, Pressure Angle, Contact Ratio, Bending Stress, Contact Stress

1. Introduction

The purpose of this study is optimisation of effective design parameters to reduce tooth bending stress for an automotive transmission gearbox.

Gears are mechanically transmitted power in automotive transmissions. Therefore, determining the geometric design parameters of gears is crucial.

By optimising all the geometric parameters of the gears, obtaining desired gearbox structures can be possible.

All constraints are also satisfied by the optimised geometric design parameters, based on pressure angle.

By optimising the effective geometric design parameters of the five-speed gearbox, such as the module, number of teeth, etc., reducing the tooth bending stress is possible.

Increasing the contact ratio results in reduced tooth bending stress and tooth contact stress. However, increased the pressure angle causes increasing of the tooth bending stress and tooth contact stress, since the contact ratio reduces depending on increasing of the pressure angle. Furthermore, higher contact ratio has a positive effect on reducing tooth bending stress. In contrast, higher pressure angle has a negative effect on reducing tooth bending stress. Application of tooth profile modification has a positive effectiveness on reducing the tooth bending stress.

The following discussion summarises findings from the literature:

1.1. Literature Review

The following results on tooth bending strength are presented in the literature:

An asymmetric gear pair improves the tooth-root bending load carrying capacity of the pinion and wheel gear at higher pressure angles on the coast side compared to a conventional symmetric gear. The optimum profile shift values increases with an increase in the speed ratio and number of teeth in the pinion, and increasing the asymmetric factor and pressure angles on the drive side improves the tooth-root bending capacity. When the speed ratio increases, the optimum maximum fillet stress increases very slightly compared to that of optimum profile shift factor for pinion [1].

Asymmetric involute-type teeth were studied, since the non-involute teeth application has a number of disadvantages. The concept of one-sided involute asymmetric spur gear teeth is to increase the load carrying capacity of the driving involute. The literature concludes that the load carrying capacity can increase to 28% higher than that of standard 20˚ involute teeth [2].

The advantage of using proposed asymmetric design in gearboxes is increased bending strength, pitting resistance, without changing the dimension or number of teeth in the gearbox [2].

An alternative method to increase the tooth bending strength of involute gear teeth is positive modification of addendum (positive shifting) the pinion and, in some cases, mating wheel. This method produces well-running teeth, but both the pitting resistance and scoring resistance are reduced due to the positive shifting [2].

A smaller pressure angle causes to produce undercut for a given number of teeth. However, the contact ratio increases, and load carrying capacity may be improved [3].

Tooth profile modification is an effective parameter for optimising the geometric design parameters of gears. A numerical study found that the application of positive profile modification results in reduced tooth bending stress and increased safety factor for tooth bending stress [4].

1.2. Gearbox Mechanism

The gearbox mechanism is shown in Figure 1. Where Z_{1p}, Z_{2p}, Z_{3p} and Z_{4p} denotes the 1st speed pinion gear, the 2nd speed pinion gear, the 3rd speed pinion gear and the 4th speed pinion gear respectively. Z_{Cp} and Z_{Rp} denote the constant speed pinion gear and the rear speed pinion gear. Z_{g1}, Z_{g2}, Z_{g3} and Z_{g4} denotes the 1st speed wheel gear, the 2nd speed wheel gear, the 3rd speed wheel gear and the 4th speed wheel gear respectively. Z_{Cg} and Z_{Rg} denote the constant speed wheel gear and the rear speed wheel gear. S_1, S_2 and S_3 denote synchronisers.

2. Effective Geometric Design Parameters

General definitions and specification factors for gears are given in DIN 868 as follows.

The module, m is the basic parameter for the linear dimensions of gear tooth systems. It is the result of dividing the pitch, p by the number π. The pitch is determined by the dimensions of the datum surface and the number of teeth; see Figure 2(a).

The number of teeth, z of a gear is the number of teeth present on the full circumference of the gear or the number that would be feasible for a chosen pitch; see Figure 2(b).

The face width, b, is the distance between the two end surfaces of the gear tooth system; see Figure 2(c).

The helix angle, β, is the angle between the helix line and horizontal axis; see Figure 2(d).

The centre distance, a, of a gear pair with parallel axes is the shortest distance between the two axes; see Figure 2(e).

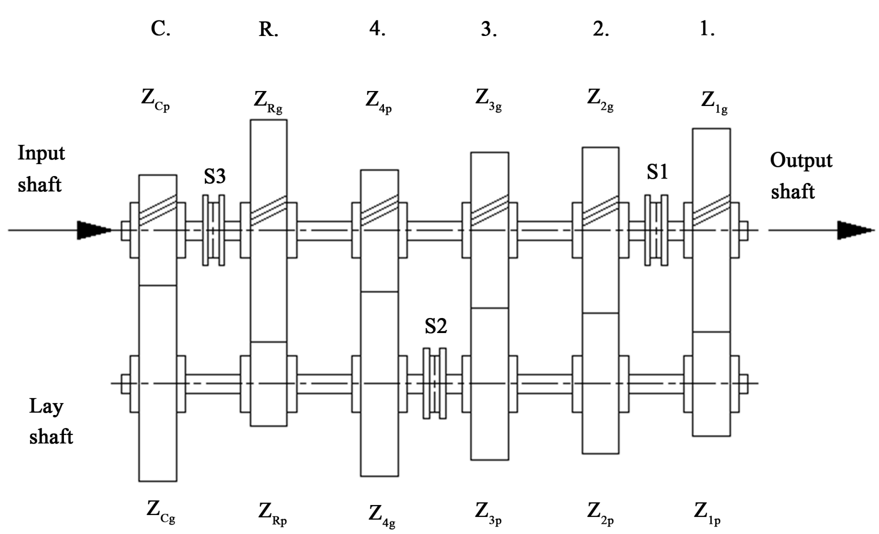

Figure 1. Five-Speed manual gearbox with helical gear for automotive transmission.

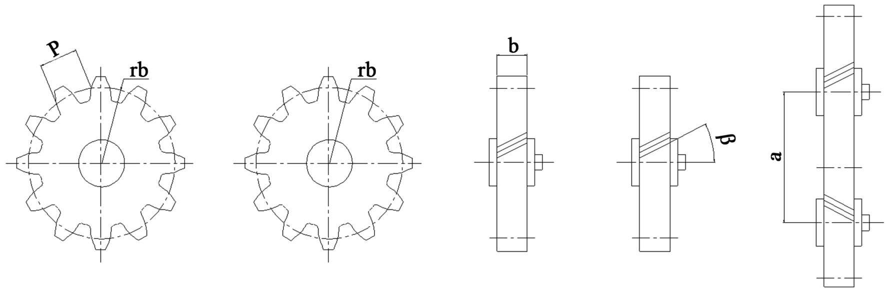

Figure 2. Design parameters for a gearbox. (a) module; (b) number of teeth z; (c) face-width b; (d) helix angle β; (e) centre distance a.

3. Contact Ratio

The dimensions of helical gear are shown in Figure 3 and the contact line is shown Figure 4. Obviously, tooth profiles must be proportioned such that a second pair of mating teeth comes into contact before the first pair is out of contact [5].

If the gear contact ratio equal to 1, then one tooth is leaving contact just as the next is beginning contact. A unity contact angle is undesirable, because slight errors in tooth spacing will cause oscillations in velocity, and, subsequently, vibration, and noise. In addition, the load will be applied on the tip of the tooth, creating the largest possible bending moment [6].

In general, the higher the contact ratio, the smoother the running of the gears. When a contact ratio is equal to 2 or more means that at least two pairs of teeth are theoretically in contact currently [5].

If a profile contact ratio is lower than 2.0, is called as Low Contact Ratio (LCR), while gearing with this parameter equal to 2.0 or greater than 2.0 is called as High Contact Ratio (HCR) [5].

The contact ratio consists of two parts, such as the transverse contact ratio, ε_α, and the overlap (face contact) ratio, ε_β.

3.1. Transverse Contact Ratio, ε_α

The *contact ratio* (CR) is defined as the average number of teeth in contact during the gear rotation. The transverse contact ratio, ε_α is calculated as follows [8].

$$\varepsilon_\alpha = \frac{g_\alpha}{p_{et}} = \frac{0.5 \cdot \left(\sqrt{d_{a1}^2 - d_{b1}^2} + \sqrt{d_{a2}^2 - d_{b2}^2}\right) - a_d \div \sin\alpha_t}{\pi \cdot m_t \cdot \cos\alpha_t} \tag{1}$$

where g_α is the path length of the contact line [mm], and p_{et} is the base pitch [mm], d_{a1} is the addendum circle diameter of the pinion gear [mm], d_{b1} is the base circle diameter of the pinion gear [mm], d_{a2} is the addendum circle diameter of the wheel gear [mm], d_{b2} is the base circle diameter of the wheel gear [mm], a_d is the centre distance [mm], α_t is the transverse pressure angle [˚], and m_t is the transverse module [mm].

The addendum circle diameter of the pinion gear, d_{a1}, is calculated as follows [8].

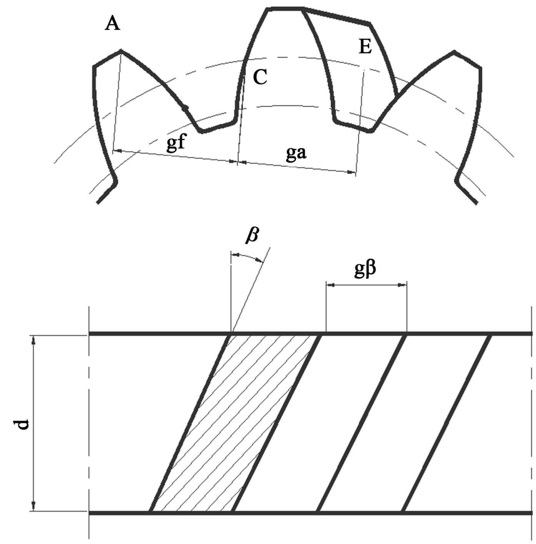

Figure 3. Contac line of helical gear.

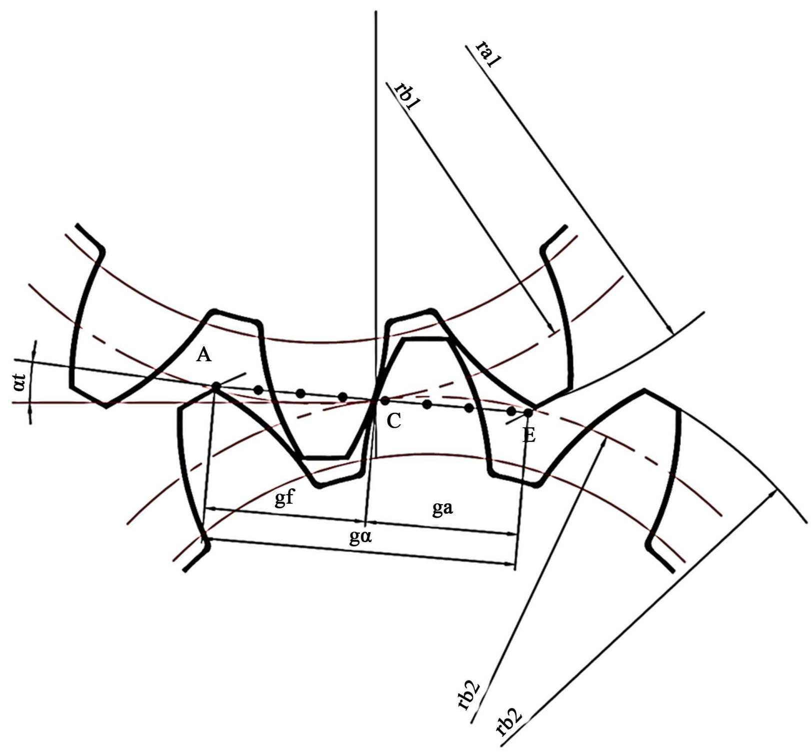

Figure 4. Contac line of helical gear including contact length AE.

$$d_{a1} = d_1 + 2 \cdot h_a = d_1 + 2 \cdot m_n = m_n \cdot \left(2 + \frac{z}{\cos \beta} \right) \tag{2}$$

where m_n is the normal module [mm], z is the number teeth [-], and β is the helix angle [˚].

The base circle diameter of the pinion gear, d_{b1}, is calculated as follows [8].

$$d_{b1} = d_1 \cdot \cos \alpha_t = z \cdot \frac{m_n}{\cos \beta} \cdot \cos \alpha_t \tag{3}$$

The addendum circle diameter of the wheel gear, d_{a2}, is calculated as follows [8].

$$d_{a2} = d_2 + 2 \cdot h_a = d_2 + 2 \cdot m_n = m_n \cdot \left(2 + \frac{z}{\cos\beta}\right) \tag{4}$$

The base circle diameter of the wheel gear, d_{b2}, is calculated as follows [9].

$$d_{b2} = d_2 \cdot \cos\alpha_t = z \cdot \frac{m_n}{\cos\beta} \cdot \cos\alpha_t \tag{5}$$

The centre distance, a_d, is calculated as follows [8].

$$a_d = \frac{d_1 + d_2}{2} = m_t \cdot \frac{(z_1 + z_2)}{2} = \frac{m_n}{\cos\beta} \frac{(z_1 + z_2)}{2} \tag{6}$$

3.2. Overlap Ratio, ε_β

The overlap ratio, ε_β is calculated as follows [8].

$$\varepsilon_\beta = \frac{U}{p_t} = \frac{b \cdot \tan\beta}{p_t} = \frac{b \cdot \sin\beta}{\pi \cdot m_n} \tag{7}$$

where U is the action length [mm], p_t is the transverse pitch [mm], b is the face width [mm], and m_n is the normal module [mm].

3.3. Total Contact Ratio, ε_γ

The total contact ratio, ε_γ is calculated as follows.

$$\varepsilon_\gamma = \varepsilon_\alpha + \varepsilon_\beta \tag{8}$$

where ε_α is the transverse contact ratio and ε_β is the overlap ratio. Helical gears have higher contact ratio than spur gears thus, they have also higher load carrying capacities than spur gears.

4. Strength of Helical Gears

The gear strength is defined by two criteria such as the tooth bending strength and tooth contact strengths according to the ISO 6336.

4.1. Tooth Bending Stress

The bending stress in distribution are shown in **Figure 5**. The real tooth-root stress, σ_F is calculated as follows [7] [8]

$$\sigma_F = \frac{F_t}{bm_n} Y_F Y_S Y_\varepsilon Y_\beta K_A K_V K_{F\beta} K_{F\alpha} \tag{10}$$

The permissible bending stress, σ_{Fp}, is calculated as follows [7] [8].

$$\sigma_{Fp} = \sigma_{F\lim} Y_{ST} Y_N Y_\delta Y_R Y_X \tag{11}$$

where all the responsible parameters for the tooth bending stress are given in **Table 1**. The safety factor for bending stress S_F is calculated as follows [7] [8]

$$S_F = \frac{\sigma_{Fp}}{\sigma_F} \tag{12}$$

Table 1. Tooth bending stress parameters.

Parameters	1st pinion	2nd pinion	3rd pinion	4th pinion	Constant pinion	Rear pinion
Torque T_L [N.mm]	392×10^3	392×10^3	316×10^3	252×10^3	200×10^3	900×10^3
gear ratio u	1.814	1.147	1.242	1.560	1	2.84
stress correction factor Y_{ST}	2	2	2	2	2	2
form factor Y_F	2.75	2.75	2.75	2.75	2.75	2.75
stress correction factor Y_S	1.60	1.60	1.60	1.60	1.60	1.60
application factor K_A	1.25	1.25	1.25	1.25	1.25	1.25
internal dynamic factor K_V	1.14	1.14	1.14	1.14	1.14	1.14
transverse load factor for tooth-root stress $K_{F\alpha}$	1.2	1.2	1.2	1.2	1.2	1.2
permissible bending stress σ_{FLim} [N/mm^2]	500	500	500	500	500	500
life factor for tooth-root stress Y_N	1	1	1	1	1	1
relative notch sensitivity factor Y_δ	1	1	1	1	1	1
relative surface factor Y_R	1	1	1	1	1	1
size factor relevant to tooth-root strength Y_X	1	1	1	1	1	1

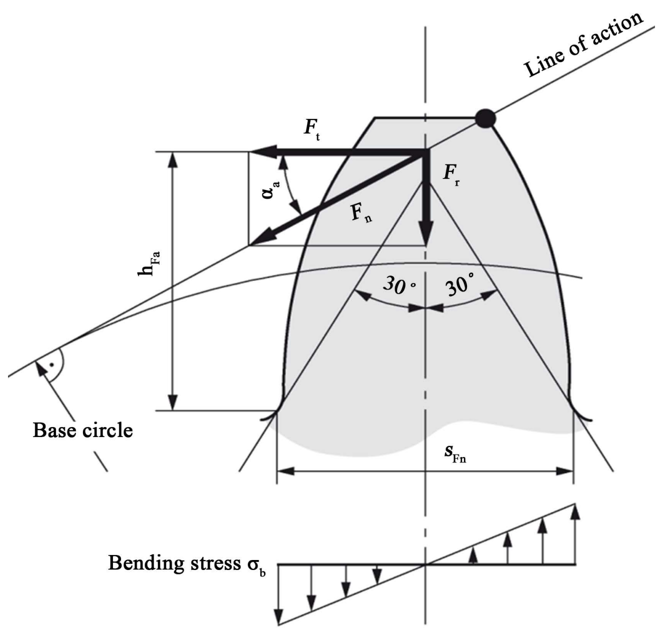

Figure 5. Bending stress at the tooth root.

4.2. Tooth Contact Stress

The contact stress, distribution is shown in Figure 6. The real contact stress, σ_H is calculated as follows [7] [8]

$$\sigma_H = \sqrt{\frac{F_t}{bm_n}\frac{u+1}{u}} Z_H Z_E Z_\varepsilon Z_\beta \sqrt{K_A K_V K_{H\beta} K_{H\alpha}} \tag{13}$$

The permissible contact stress, σ_{Hp} is calculated as follows [7] [8]:

$$\sigma_{Hp} = \sigma_{H\lim} Z_N Z_L Z_V Z_R Z_W Z_X \tag{14}$$

where all the responsible parameters for the tooth contact stress are given in Table 2.

Table 2. Tooth contact stress parameters.

Parameters	1st pinion	2nd pinion	3rd pinion	4th pinion	Constant pinion	Rear pinion
Torque T_L [N.mm]	392×10^3	392×10^3	316×10^3	252×10^3	200×10^3	900×10^3
gear ratio u	1.814	1.147	1.242	1.560	1	2.84
zone factor Z_H	1	1	1	1	1	1
elasticity factor Z_E	189.8	189.8	189.8	189.8	189.8	189.8
transverse load factor for contact stress $K_{H\alpha}$	1.2	1.2	1.2	1.2	1.2	1.2
permissible contact stress σ_{Hlim} [N/mm^2]	1400	1400	1400	1400	1400	1400
life factor for contact stress Z_N	1	1	1	1	1	1
velocity factor Z_V	1	1	1	1	1	1
roughness factor Z_R	1	1	1	1	1	1
work hardening factor Z_W	1	1	1	1	1	1
size factor for contact stress Z_X	1	1	1	1	1	1

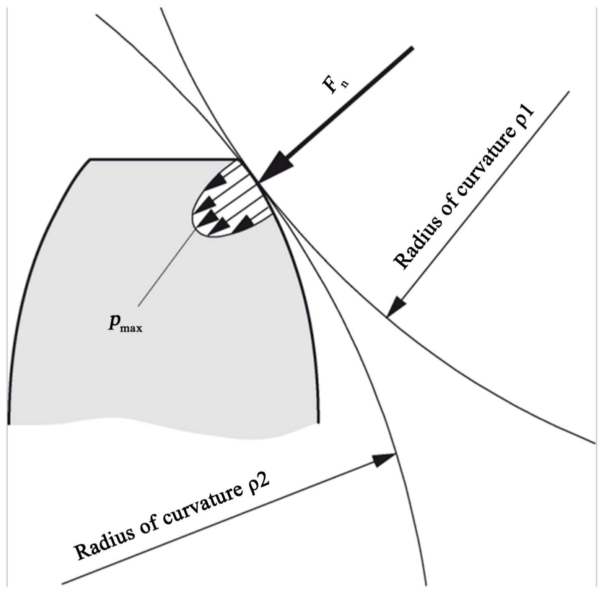

Figure 6. Contact stress at the tooth flank.

The safety factor for contact stress, S_H, is calculated as follows [7] [8]:

$$S_H = \frac{\sigma_{Hp}}{\sigma_H} \tag{15}$$

5. Optimisation of Effective Design Parameters of Gearbox

Constrained optimisation method is helpful for designing light-weight gearbox structures. Constraints, including tooth contact stress and constant distance between gear centres can be used for this optimisation.

During optimisation, the aim is typically to minimise the cost of a structure while satisfying all the design requirements. By optimising the effective design parameters, a light-weight gearbox structure design is also possible [9] [10].

5.1. Objectives Function

Tooth bending stresses are considered as objective functions, during the optimisation study. The flowchart of the optimisation procedure of geometric design parameters is shown in Figure 7. The following objective function was used:

$$F = \min(\sigma) \tag{16}$$

Following minimum tooth bending stress is defined as objective function:

$$\min(\sigma_F) = \min\left(\frac{F_t}{bm_n} Y_F Y_S Y_\varepsilon Y_\beta K_A K_V K_{F\beta} K_{F\alpha}\right) \tag{17}$$

Thus, the module m, the number of teeth z, and the helix angle β, are the design parameters to be determined. During the constrained optimisation, the following optimisation problem is solved:

$$\min \sigma(m, z, \beta, b) \tag{18}$$

$$\text{Subject to: } \mathbf{LB} \le m, z, \beta, b \le \mathbf{UB} \text{ and } G(X) \le 0 \tag{19}$$

where LB is lower bound and UB is upper bounds on the design parameter vector. The iterations start with the initial values of design parameters such as, m_0, z_0, β_0, and b_0. Initial design parameters X0 are varied during the optimisation process, where G (X) ≤ 0 is the nonlinear inequalities.

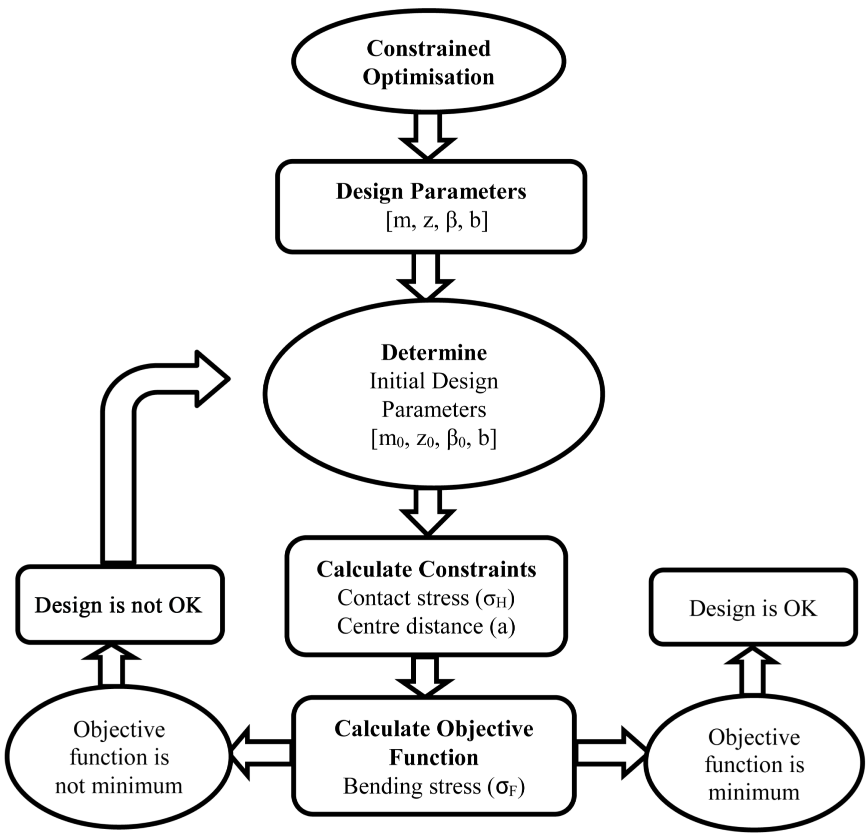

Figure 7. Flow chart to optimise gearbox design parameters.

5.2. Constraint Functions

During constraint optimisation, the tooth contact stress and constant distance between gear centres are considered as the constraint function as follows:

$$\sigma_H - \sigma_{Hp} \le 0 \tag{20}$$

where σ_H is the real contact stress [N/mm^2] and σ_{Hp} is the permissible contact stress [N/mm^2].

$$a_1 = a_2 = a_3 = a_4 = a_5 = a_R = \text{constant} \tag{21}$$

where a_1 is the centre distance of the 1st speed, a_2 is the centre distance of the 2nd speed, a_3 is the centre distance of the 3rd speed, a_4 is the centre distance of the 4th speed, a_5 is the centre distance of the 5th speed and a_R is the centre distance of the rear speed.

6. Numerical Example

Constrained optimisation method is applied to the five-speed gearbox mechanism to reduce tooth bending stress. All optimisation programs are developed using MATLAB. The sequential quadratic programming (SQP) method is used.

Twenty-four design parameters are optimised simultaneously using the developed programs. All the parameters for the tooth strength calculation are shown in Table 1 and Table 2, respectively.

7. Results

It is observed in solution 1 (Table 3) that the obtained optimum effective parameters result in satisfied values for each speed. By considering safety factors, this solution is more acceptable for 1st and rear speed. The safety factor for bending stress, S_F, ranges between 1.1797 and 3.1783, and the safety factor for contact stress, S_H, varies between 1.2269 and 2.5490.

It is observed in solution 2 (Table 3) that the obtained optimum effective parameters result in acceptable values for each speed. The safety factor for bending stress, S_F, ranges between 1.1254 and 3.0457, and the safety factor for contact stress, S_H, varies between 1.1854 and 2.4725.

The results from solution 3 (Table 3) show that the obtained optimum effective parameters satisfy desired requirements. By considering safety factors, this solution is more acceptable for 2nd and 3rd speed. The safety factor for bending stress, S_F, ranges between 1.0776 and 2.9275, and the safety factor for contact stress, S_H, varies between 1.1491 and 2.4046.

The results from solution 4 (Table 3) indicate that the obtained optimum effective parameters satisfy all requirements. The safety factor for bending stress, S_F, ranges between 1.0357 and 2.8229, and the safety factor for contact stress, S_H, varies between 1.1175 and 2.3448.

The results from solution 5 (Table 3) show that the obtained optimum solutions result in desired values. However, by considering safety factors, this solution is more acceptable for constant pinion. The safety factor for bending stress, S_F, ranges between 0.9993 and 2.7314, and the safety factor for contact stress, S_H, varies between 1.0901 and 2.2926.

Table 3. (a) Optimisation results-Solution no 1; (b) Optimisation results-Solution no 2; (a) Optimisation results-Solution no 3; (d) Optimisation results-Solution no 4; (e) Optimisation results-Solution no 5; (f) Optimisation results-Solution no 6.

(a)

Solution no 1 (pressure angle α = 12˚)
Lower bound Lb =[2 2 2 2 2 2 14 19 19 19 19 19 20 20 20 20 20 30 30 30 30 30 30 40]
Upper bound Ub = [7 7 7 7 7 7 14 19 19 19 19 19 32 32 32 32 34 34 34 34 34 34 42]
Initial condition X0 = [7 7 7 7 7 7 14 19 19 19 19 19 31 31 31 31 31 32 33 33 32 32 32 42]

	1st pinion	2nd pinion	3rd pinion	4th pinion	Constant pinion	Rear pinion
Module m	4.4442	3.3709	3.2283	2.8281	3.6180	2.7141
Number of teeth z	14.000	19.000	19.000	19.000	19.000	19.000
Helix angle β	32.000	30.7483	30.7391	30.7133	30.7645	31.6943
Face width b	34.000	33.000	32.000	32.000	32.000	44.000
Pressure angle α_t	14.0709	13.8919	13.8906	13.8871	13.8942	14.0261
Centre distance a	80.000	80.000	80.000	80.000	80.000	80.000
Transverse contact ratio ε_α	1.7836	1.8255	1.8423	1.8895	1.7963	1.8913
Overlap ratio ε_β	1.6651					
Bending stress σ_F	822.2394	679.0009	692.3186	558.4413	314.6310	847.6631
Safety factor for bending stress S_F	1.2162	1.4728	1.4444	1.7907	3.1783	1.1797
Contact stress σ_H	921.600	780.1000	726.1000	697.2000	549.2000	1141.100
Safety factor for contact stress σ_H	1.5191	1.7946	1.9280	2.0081	2.5490	1.2269

(b)

Solution no 2 (pressure angle α = 14˚)
Lower bound Lb = [as same as above]
Upper bound Ub = [as same as above]
Initial condition X0 = [as same as above]

	1st pinion	2nd pinion	3rd pinion	4th pinion	Constant pinion	Rear pinion
Module m	3.4442	3.3708	3.2282	2.8280	3.6179	2.7140
Number of teeth z	14.000	19.000	19.000	19.000	19.000	19.000
Helix angle β	32.000	30.7513	30.7423	30.7172	30.7671	31.6986
Face width b	34.000	33.000	32.000	32.000	32.000	44.0000
Pressure angle α_t	16.3835	16.1785	16.1771	16.1731	16.1810	16.3329
Centre distance a	80.000	80.000	80.000	80.000	80.000	80.000
Transverse contact ratio ε_α	1.6768	1.7140	1.7277	1.7656	1.6901	1.7655
Overlap ratio ε_β	1.6651					
Bending stress σ_F	858.9134	709.4125	723.8493	585.0816	328.3350	888.5450
Safety factor for bending stress S_F	1.1643	1.4096	1.3815	1.7092	3.0457	1.1254
Contact stress σ_H	950.500	805.100	749.800	721.200	566.200	1181.000
Safety factor for contact stress σ_H	1.4729	1.7389	1.8671	1.9412	2.4725	1.1854

(c)

Solution no 3 (pressure angle α = 16˚) Lower bound Lb = [as same as above] Upper bound Ub = [as same as above] Initial condition X0 = [as same as above]						
	1st pinion	2nd pinion	3rd pinion	4th pinion	Constant pinion	Rear pinion
Module m	3.4442	3.3707	3.2281	2.8278	3.6178	2.7138
Number of teeth z	14.000	19.000	19.000	19.000	19.000	19.000
Helix angle β	32.000	30.7541	30.7453	30.7209	307695	31.7026
Face width b	34.000	33.000	32.000	32.000	32.000	44.000
Pressure angle α_t	18.6816	18.4523	18.4507	18.4463	18.4550	18.6256
Centre distance a	80.000	80.000	80.000	80.000	80.000	80.000
Transverse contact ratio ε_α	1.5855	1.6184	1.6296	1.6603	1.5986	1.6589
Overlap ratio ε_β	1.6651					
Bending stress σ_F	894.2161	738.8489	754.3754	610.8693	341.5908	928.0272
Safety factor for bending stress S_F	1.1183	1.3535	1.3256	1.6370	2.9275	1.0776
Contact stress σ_H	977.500	828.500	772.000	743.700	582.200	1218.400
Safety factor for contact stress σ_H	1.4323	1.6897	1.8134	1.8824	2.4046	1.1491

(d)

Solution no 4 (pressure angle α = 18˚) Lower bound Lb = [as same as above] Upper bound Ub = [as same as above] Initial condition X0 = [as same as above]						
	1st pinion	2nd pinion	3rd pinion	4th pinion	Constant pinion	Rear pinion
Module m	3.4442	3.3703	3.2278	2.8275	3.6175	2.7137
Number of teeth z	14.000	19.000	19.000	19.000	19.000	19.000
Helix angle β	32.000	30.7565	30.7478	30.7240	30.7716	31.7060
Face width b	34.000	33.000	32.000	32.000	32.000	44.000
Pressure angle α_t	20.9637	20.7116	20.7099	20.7052	20.7146	20.9028
Centre distance a	80.000	79.9942	79.9938	79.9922	79.9950	79.9996
Transverse contact ratio ε_α	1.5077	1.5367	1.5459	1.5709	1.5202	1.5688
Overlap ratio ε_β	1.6651					
Bending stress σ_F	927.6637	766.9720	783.5646	635.5152	354.2463	965.5743
Safety factor for bending stress S_F	1.0780	1.3038	1.2762	1.5735	2.8229	1.0357
Contact stress σ_H	1000.240	850.300	792.700	764.600	597.100	1252.90
Safety factor for contact stress σ_H	1.3967	1.6464	1.7661	1.8310	2.3448	1.1175

(e)

Solution no 5 (pressure angle α = 20˚) Lower bound Lb = [as same as above] Upper bound Ub = [as same as above] Initial condition X0 = [as same as above]						
	1st pinion	2nd pinion	3rd pinion	4th pinion	Constant pinion	Rear pinion
Module m	3.4442	3.3699	3.2274	2.8270	3.6172	2.7136
Number of teeth z	14.000	19.000	19.000	19.000	19.000	19.000
Helix angle β	32.000	30.7584	30.7499	30.7265	30.7733	31.7088
Face width b	34.000	33.000	32.000	32.000	32.000	44.000
Pressure angle α_t	23.2283	22.9551	22.9533	22.9483	22.9583	23.1628
Centre distance a	80.000	79.9868	79.9857	79.9820	79.9885	79.9990
Transverse contact ratio ε_α	1.4417	1.4672	1.4749	1.4955	1.4535	1.4930
Overlap ratio ε_β	1.6651					
Bending stress σ_F	958.800	793.400	811.000	658.700	366.100	1000.700
Safety factor for bending stress S_F	1.0429	1.2604	1.2331	1.5182	2.7314	0.9993
Contact stress σ_H	1025.100	870.300	811.600	783.800	610.700	1284.300
Safety factor for contact stress σ_H	1.3658	1.6087	1.7249	1.7862	2.2926	1.0901

(f)

Solution no 6 (pressure angle α = 22˚) Lower bound Lb = [as same as above] Upper bound Ub = [as same as above] Initial condition X0 = [as same as above]						
	1st pinion	2nd pinion	3rd pinion	4th pinion	Constant pinion	Rear pinion
Module m	3.4442	3.3696	3.2271	2.8267	3.6169	2.7136
Number of teeth z	14.000	19.000	19.000	19.000	19.000	19.000
Helix angle β	32.000	30.7598	30.7514	30.7284	30.7745	31.7109
Face width b	34.000	33.000	32.000	32.000	32.000	44.000
Pressure angle α_t	25.4740	25.1815	25.1796	25.1743	25.1849	25.4043
Centre distance a	80.000	79.9806	79.9790	79.9735	79.9831	79.9988
Transverse contact ratio ε_α	1.3862	1.4086	1.4150	1.4321	1.3970	1.4295
Overlap ratio ε_β	1.6651					
Bending stress σ_F	987.300	817.700	836.200	680.000	377.000	1033.100
Safety factor for bending stress S_F	1.0128	1.2230	1.1958	1.4706	2.6522	0.9680
Contact stress σ_H	1045.400	888.300	828.700	801.000	622.900	1312.500
Safety factor for contact stress σ_H	1.3392	1.5761	1.6894	1.7478	2.2475	1.0667

The results from solution 6 (Table 3) indicate that the obtained optimum values satisfy all requirements. However, by considering safety factors, this solution is more acceptable for 4th speed. The safety factor for bending stress, S_F, ranges between 0.9680 and 2.6522, and the safety factor for contact stress, S_H, varies between 1.0667 and 2.2475.

From the obtained optimisation results, it can be concluded that increasing the contact ratio results in reduced tooth bending stress and reduced contact stress. Furthermore, increased the pressure angle caused increased the tooth bending stress and contact stress, by reducing the contact ratio. The relations between the contact ratio and bending stress are shown in Figures 8-13. The contact ratio and pressure angle relations are shown in Figures 14-19.

7.1. Contact Ratio and Tooth Bending Stress Relation

The contact ratio and bending stress relation for the 1st speed is shown in Figure 8. As the contact ratio increases from 1.3862 to 1.7836, the bending stress reduces from 987.300 [N/mm^2] to 822.2394 [N/mm^2]. Thus, increasing the contact ratio 28.66% results in a 20.07% reduction in tooth bending stress.

The contact ratio and bending stress relation for the 2nd speed is shown in Figure 9. As the contact ratio for the 2nd speed increases from 1.4086 to 1.8255, the bending stress reduces from 817.7000 [N/mm^2] to 679.0009 [N/mm^2]. Thus, increasing the contact ratio 29.59% reduces the tooth bending stress 20.42%.

The contact ratio and bending stress relation for the 3rd speed is shown in Figure 10. As the contact ratio for the 3rd speed increases from 1.4150 to 1.8423, bending stress reduces from 836.2000 [N/mm^2] to 692.3186 [N/mm^2]. Thus, increasing the contact ratio 30.19%, results a 20.78% reduction in tooth bending stress.

The contact ratio and bending stress relation for the 4th speed is shown in Figure 11. As the contact ratio for the 4th speed increases from 1.4321 to 1.8895, the bending stress reduces from 680.0000 [N/mm^2] to 558.4413 [N/mm^2]. Thus, increasing the contact ratio 31.93% reduces the tooth bending stress 21.76%.

The contact ratio and bending stress relation for the 5th speed is shown in Figure 12. As the contact ratio for the 5th speed increases from 1.3970 to 1.7963, the bending stress reduces from 377.0000 [N/mm^2] to 314.6310 [N/mm^2]. Thus, increasing the contact ratio 28.58%, results in a 19.82% reduction in the tooth bending stress.

The contact ratio and bending stress relation for the rear speed is shown in Figure 13. As the contact ratio for the rear speed increases from 1.4295 to 1.8913, the bending stress reduces from 1033.1000 [N/mm^2] to 847.6631 [N/mm^2]. Thus, increasing the contact ratio 32.30%, reduces the tooth bending stress 21.87%.

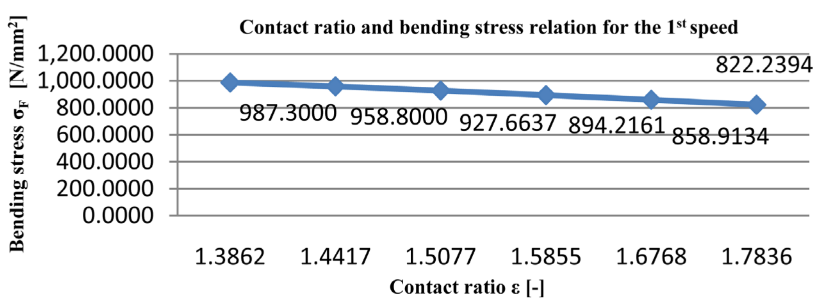

Figure 8. Contact ratio and bending stress relation for the 1st speed.

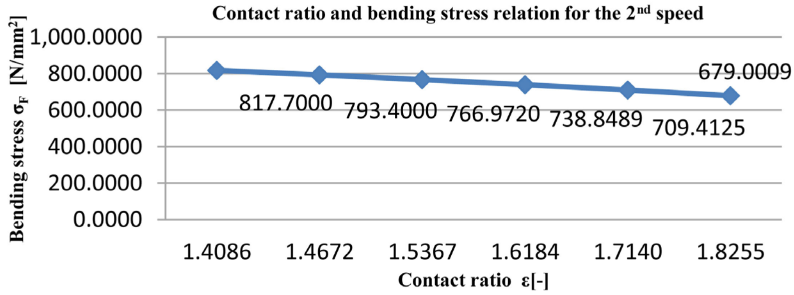

Figure 9. Contact ratio and bending stress relation for the 2nd speed.

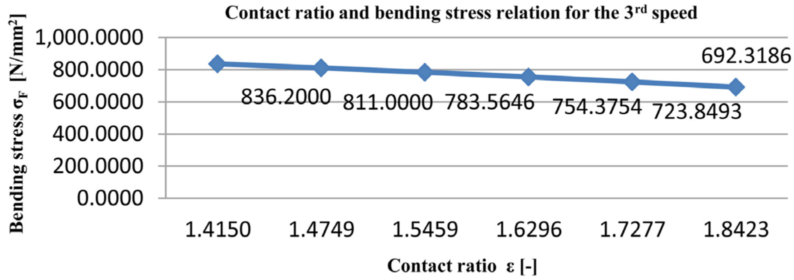

Figure 10. Contact ratio and bending stress relation for the 3rd speed.

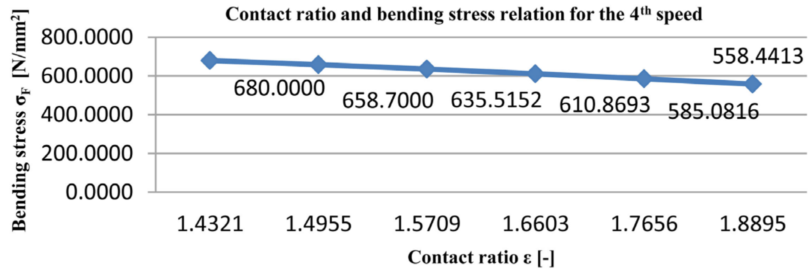

Figure 11. Contact ratio and bending stress relation for the 4th speed.

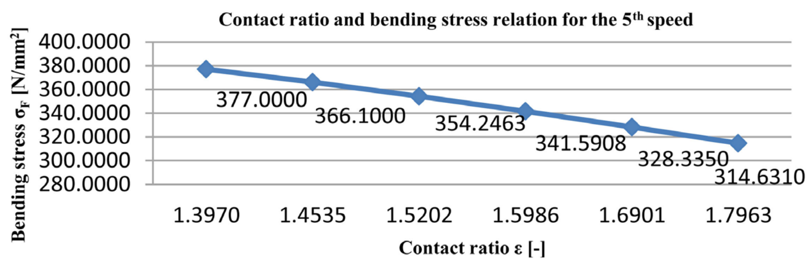

Figure 12. Contact ratio and bending stress relation for the 5th speed.

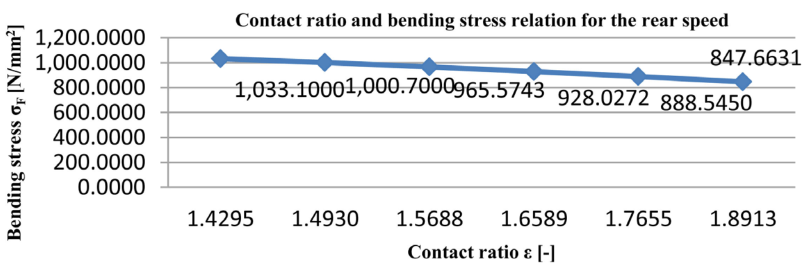

Figure 13. Contact ratio and bending stress relation for the rear speed.

7.2. Contact Ratio and Pressure Angle Relation

The contact ratio and pressure angle relation for the 1st speed is shown in Figure 14. As the pressure angle for the 1st speed reduces from 22 [˚] to 12 [˚], the contact ratio increases from 1.3862 to 1.7836. Thus, decreasing the pressure angle 83%, results in a 28.66% increase in the contact ratio.

The contact ratio and pressure angle relation for the 2nd speed is shown in Figure 15. As the pressure angle for the 2nd speed reduces from 22 [˚] to 12 [˚], the contact ratio increases from 1.4086 to 1.8255. Thus, decreasing the pressure angle 83%, increases the contact ratio 29.59%.

The contact ratio and pressure angle relation for the 3rd speed is shown in Figure 16. As the pressure angle for the 3rd speed reduces from 22 [˚] to 12 [˚], the contact ratio increases from 1.4150 to 1.8423. Thus, decreasing the pressure angle 83%, results in a 30.19% increase in the contact ratio.

The contact ratio and pressure angle relation for the 4th speed is shown in Figure 17. As the pressure angle for the 4th speed reduces from 22 [˚] to 12 [˚], the contact ratio increases from 1.4321 to 1.8895. Thus, decreasing the pressure angle 83%, result in increases the contact ratio 31.93%.

The contact ratio and pressure angle relation for the 5th speed is shown in Figure 18. As the pressure angle for the 5th speed reduces from 22 [˚] to 12 [˚], the contact ratio increases from 1.3970 to 1.7963. Thus, decreasing the pressure angle 83%, results in a 28.58% increase in the contact ratio.

The contact ratio and pressure angle relation for the rear speed is shown in Figure 19. As the pressure angle for the rear speed reduces from 22 [˚] to 12 [˚], the contact ratio increases from 1.4295 to 1.8913. Thus, decreasing the pressure angle 83%, results in a 32.30% increase in the contact ratio.

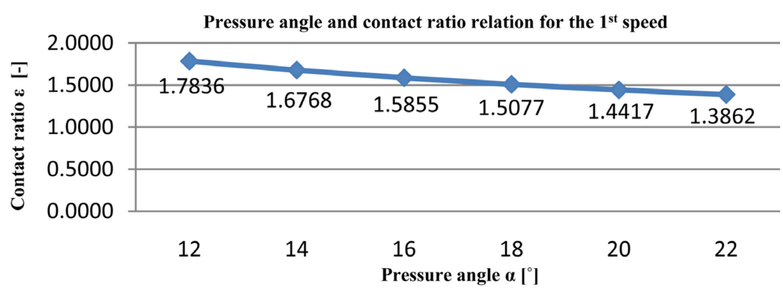

Figure 14. Contact ratio and pressure angle relation for the 1st speed.

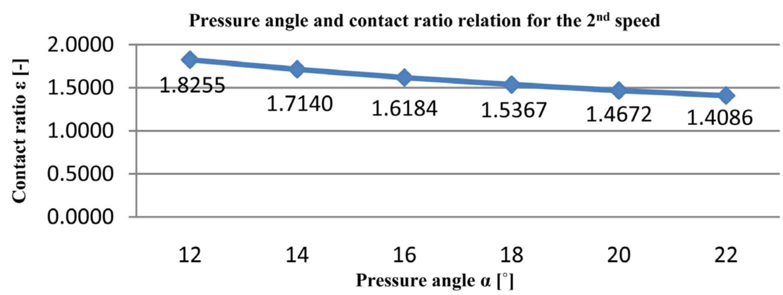

Figure 15. Contact ratio and pressure angle relation for the 2nd speed.

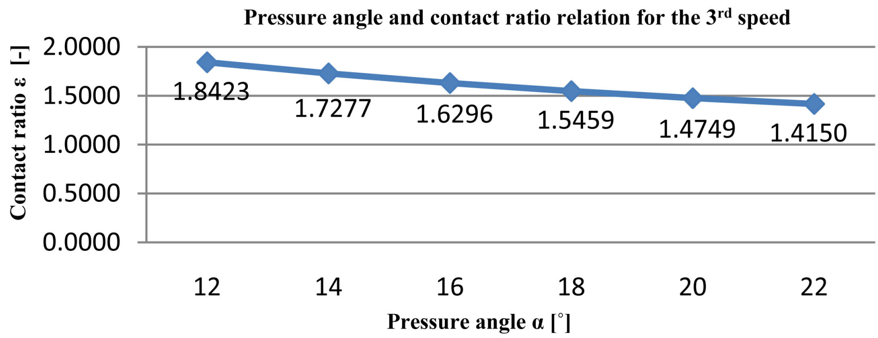

Figure 16. Contact ratio and pressure angle relation for the 3rd speed.

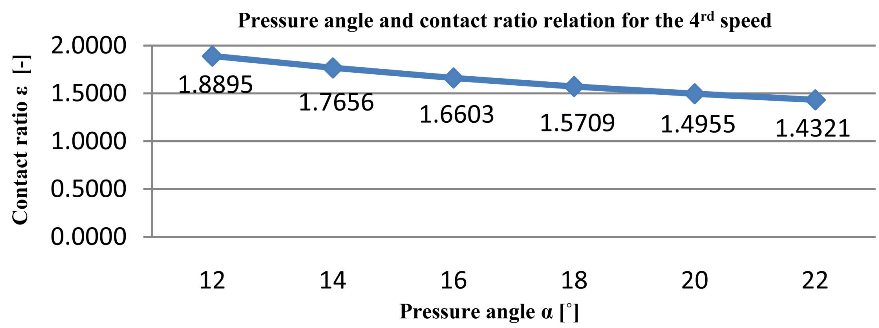

Figure 17. Contact ratio and pressure angle relation for the 4th speed.

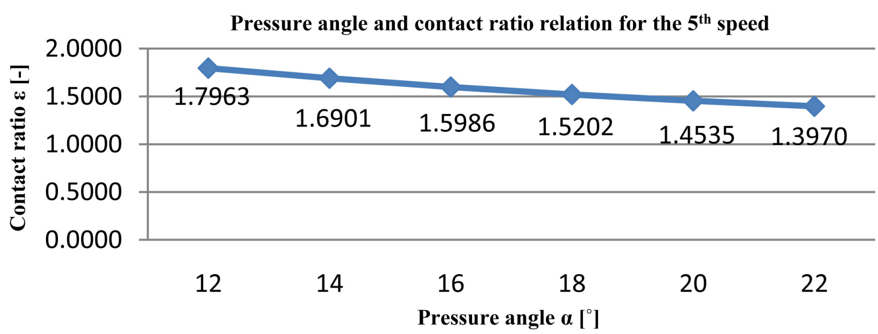

Figure 18. Contact ratio and pressure angle relation for the 5th speed.

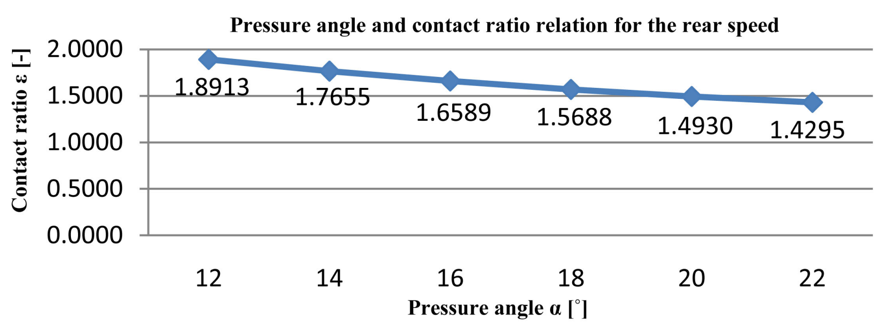

Figure 19. Contact ratio and pressure angle relation for the rear speed.

7.3. Tooth Profile Modification Factor and Bending Stress Relation

The tooth profile modification and bending stress relation for the 1st speed is shown in Figure 20. While the profile modification factor increase from 0 to 0.3, the bending stress reduces from 1171.5000 [N/mm^2] to 927.4486 [N/mm^2].

The tooth profile modification and bending stress relation for the 2nd speed is shown in Figure 21. As the profile modification factor increase from 0 to 0.3, the bending stress reduces from 854.7000 [N/mm^2] to 712.5610 [N/mm^2].

The tooth profile modification and bending stress relation for the 3rd speed is shown in Figure 22. While the profile modification factor increase from 0 to 0.3, the bending stress reduces from 873.9000 [N/mm^2] to 728.3622 [N/mm^2].

The tooth profile modification and bending stress relation for the 4rd speed is shown in Figure 23. As the profile modification factor increase from 0 to 0.3, the bending stress reduces from 709.5000 [N/mm^2] to 591.5892 [N/mm^2].

The tooth profile modification and bending stress relation for the 5th speed is shown in Figure 24. As the profile modification factor increase from 0 to 0.3, the bending stress reduces from 394.4000 [N/mm^2] to 328.8225 [N/mm^2].

The tooth profile modification and bending stress relation for the rear speed is shown in Figure 25. As the profile modification factor increase from 0 to 0.3, the bending stress reduces from 1074.800 [N/mm^2] to 899.1084 [N/mm^2].

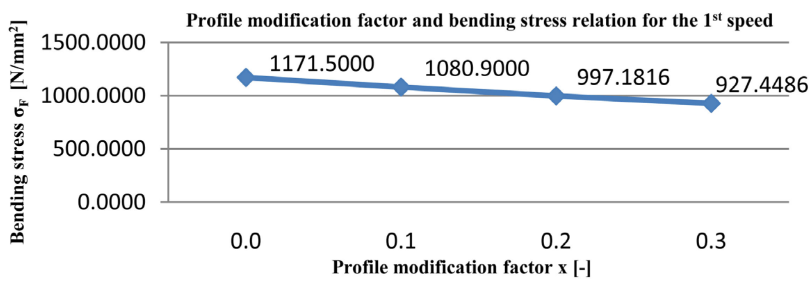

Figure 20. Profile modification factor and bending stress relation for the 1st speed.

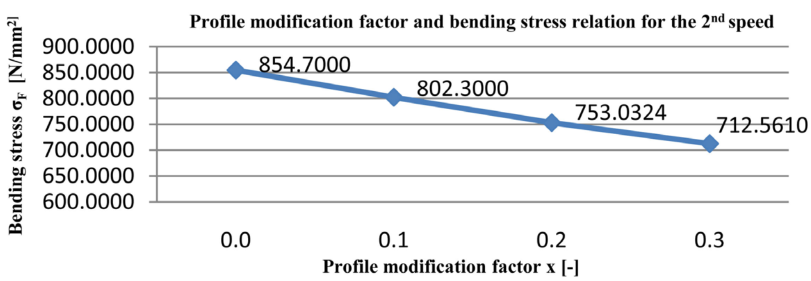

Figure 21. Profile modification factor and bending stress relation for the 2nd speed.

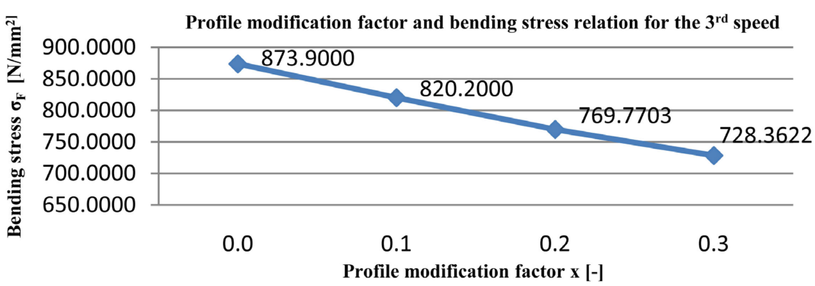

Figure 22. Profile modification factor and bending stress relation for the 3rd speed.

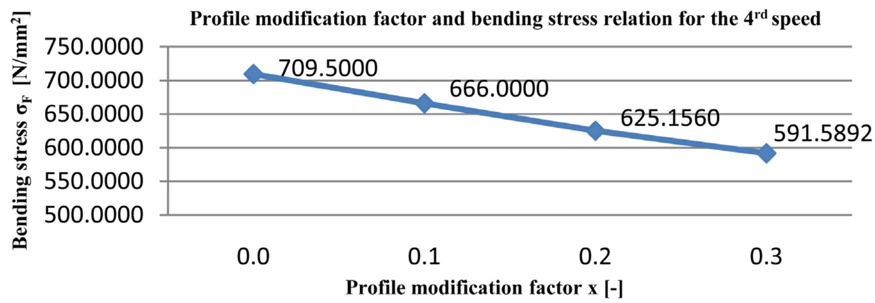

Figure 23. Profile modification factor and bending stress relation for the 4th speed.

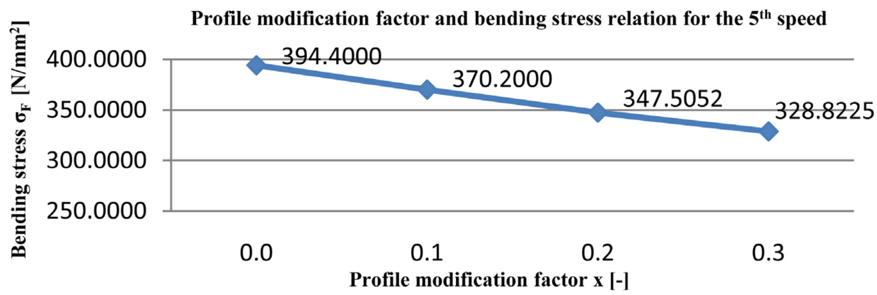

Figure 24. Profile modification factor and bending stress relation for the 5th speed.

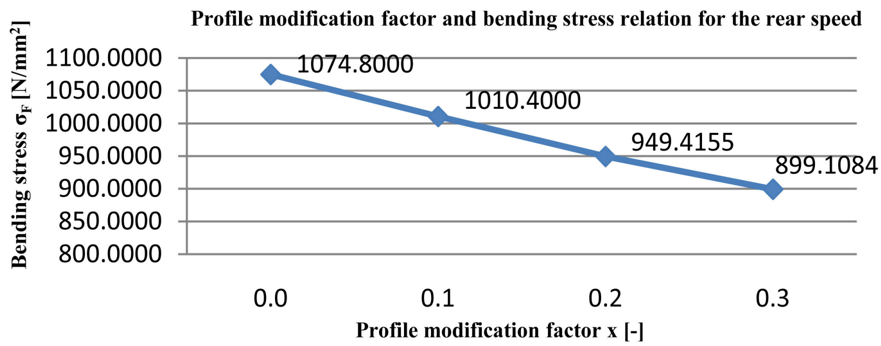

Figure 25. Profile modification factor and bending stress relation for the rear speed.

7.4. Optimum Design of Effective Parameters

A flowchart of the optimum design of effective parameters based on pressure angle is shown in Figure 26.

The safety factor for bending stress, S_F, and safety factor for contact stress S_H, are the basic selection criteria used by the Optimum Design. The Selective Optimum Design is shown in Table 4.

Although, obtained optimised geometric design parameters are significant for all constraints, the best solutions, based on pressure angle are determined from the obtained optimum solutions for each speed.

The geometric design parameters are optimised simultaneously for each given gearbox speed. However, it is not necessary to choose a single solution that changes with respect to the pressure angle. Therefore, all effective geometric design parameters can be determined independently for each speed from obtained optimum solutions.

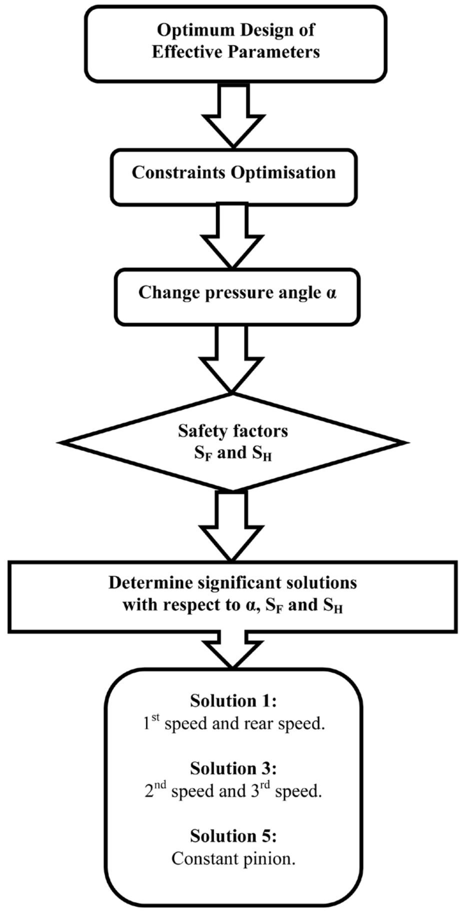

Figure 26. Flowchart of optimum design of effective parameters.

Table 4. Determination of best optimum solution.

Lower bound Lb = [2 2 2 2 2 2 14 19 19 19 19 19 20 20 20 20 20 30 30 30 30 30 30 40]
Upper bound Ub = [7 7 7 7 7 7 14 19 19 19 19 19 32 32 32 32 34 34 34 34 34 34 34 42]
Initial condition X0 = [7 7 7 7 7 7 14 19 19 19 19 19 31 31 31 31 31 32 33 33 32 32 32 42]

	Sol.1 Pressure angle $\alpha = 12^\circ$	Sol.3 Pressure angle $\alpha = 16^\circ$	Sol.3 Pressure angle $\alpha = 16^\circ$	Sol.3 Pressure angle $\alpha = 22^\circ$	Sol.5 Pressure angle $\alpha = 20^\circ$	Sol.1 Pressure angle $\alpha = 12^\circ$
	1st pinion	2nd pinion	3rd pinion	4th pinion	Constant pinion	Rear pinion
Module m	4.4442	3.3707	3.2281	2.8267	3.6172	2.7141
Number of teeth z	14.000	19.000	19.000	19.000	19.000	19.000
Helix angle β	32.000	30.7541	30.7453	30.7284	30.7733	31.6943
Face width b	34.000	33.000	32.000	32.000	32.000	44.000
Pressure angle α_t	14.0709	18.4523	18.4507	25.1743	22.9583	14.0261
Centre distance a	80.000	80.000	80.000	79.9735	79.9885	80.000
Transverse contact ratio ε_α	1.7836	1.6184	1.6296	1.4321	1.4535	1.8913
Overlap ratio ε_β	1.6651					
Bending stress σ_F	822.2394	738.8489	754.3754	680.000	366.100	847.6631
Safety factor for bending stress S_F	1.2162	1.3535	1.3256	1.4706	2.7314	1.1797
Contact stress σ_H	921.600	828.500	772.000	801.000	610.700	1141.100
Safety factor for contact stress σ_H	1.5191	1.6897	1.8134	1.7478	2.2926	1.2269

8. Conclusions

Optimisation of effective design parameters to reduce tooth bending stress for an automotive transmission gearbox is presented. The tooth bending stress is considered as the objective function, and the geometric design parameters are optimized under two different constraints. Tooth contact stress and constant distance between gear centres are considered as the constraints function. During optimization study, pressure angles were varied, thus contact ratios were also changed with respect to the pressure angle. The effect of the contact ratio on the tooth bending stress is analysed, and the following conclusions are drawn:

By optimising the effective geometric design parameters of the five-speed gearbox, such as the module, number of teeth, etc., reducing the tooth bending stress is possible.

Increasing the contact ratio results in reduced tooth bending stress and tooth contact stress. However, increased the pressure angle causes increasing of the tooth bending stress and tooth contact stress, since the contact ratio reduces depending on increasing of the pressure angle. Furthermore, higher contact ratio has a positive effect on reducing tooth bending stress. In contrast, higher pressure angle has a negative effect on reducing tooth bending stress. Application of tooth profile modification has a positive effectiveness on reducing the tooth bending stress.

Increasing the contact ratio 28.58% - 32.30%, results in a 19.82% - 21.87% reduction in tooth bending stress. In contrast, decreasing the pressure angle 83%, increases the contact ratio 28.58% - 32.30%. Gears with having higher contact ratio, have higher load carrying capacities.

Although, all the determined optimised geometric design parameters satisfy all constraints, it is not necessary to choose a single solution that changes with respect to the pressure angle.

All effective geometric design parameters can be determined independently for each speed inside the obtained optimum solutions. Based on pressure angle, the best optimised solutions are determined from the obtained optimum solutions for each speed in five-speed gearbox.

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