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Estimate of Population Variance by Introducing Gini's Mean Difference as Auxiliary Information

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Authors' contributions

This work was carried out in collaboration between all authors. Author MAB designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Authors SM, SAM, TAR and AR managed the analyses of the study. Authors NAS and IAS managed the literature searches. All authors read and approved the final manuscript.

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ABSTRACT

In this study we have introduced a new reliable and robust ratio estimator for estimating the population variance by utilizing the auxiliary information as Gini's mean difference. Efficiency conditions along with bias and mean square error has been worked out through a numerical demonstration under which proposed estimator have proven better than the existing estimators under consideration.

Keywords: Ratio estimator; Gini's mean difference; SRSWOR; MSE; bias and efficiency.

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1. INTRODUCTION

Here we consider a finite population $U = \{U_1, U_2, ..., U_N\}$ of N distinct and identifiable units. Let Y be a real variable with value Y_i measured on $U_i, i = 1,2,3.....N$ given a vector $[Y_1, Y_2, Y_3.........Y_N]$. Sometimes in sample surveys information on auxiliary variable X correlated with study variable Y, is available can be utilized to obtain the efficient estimator for the estimation of Population variance. "Here in this modification our aim is to estimate the population variance $S_y^2 = \frac{1}{N-1}\sum_{i-1}^{N}(y_i - \bar{y})^2$ on the bases of random sample selected from population U" to obtain the improved and precise estimates. Various authors have proposed different ratio estimators for finite population variance by utilizing the ancillary information of different parameters of population such as Isaki [1], Kadilar and Cingi [2], Subramani and Kumarapandiyan [3] and Bhat et al. [4].

2. MATERIALS AND METHODS

2.1 Notations

$N =$ Population size. $n =$ sample size. $\gamma = \frac{1}{n}$,Y= study variable. X= Auxiliary variable. $\bar{X}$, $\bar{Y}$ = Population means. $\bar{x}$, $\bar{y} =$ Sample means. S_Y^2 , $S_x^2 =$ Population variances. s_y^2 , s_x^2 = sample variances. C_x , $C_y =$ Coefficient of variation. $\rho =$ Correlation coefficient. $\beta_{1(x)} =$ Skewness of the auxiliary variable. $\beta_{2(x)} =$ Kurtosis of the auxiliary variable. $\beta_{2(y)} =$ Kurtosis of the study variable. B (.)=Bias of the estimator.
MSE(.)= Mean square error. $\hat{S}_R^2 =$ Ratio type variance estimator. G = Gini's Mean Difference.

3. Suggested Estimator

$$\hat{S}_{MS1}^2 = s_y^2\left[\frac{S_x^2 + G}{s_x^2 + G}\right]$$

We have derived here the bias and mean square error of the proposed estimator to first order of approximation as given below:

Let $e_0 = \frac{s_y^2 - S_y^2}{S_y^2}$ and $e_1 = \frac{s_x^2 - S_x^2}{S_x^2}$. Further we can write and $s_y^2 = S_y^2(1+e_0)$ $s_x^2 = S_x^2(1+e_1)$ and from the definition of e_0 and e_1 we obtain:

$E[e_0] = E[e_1] = 0$, $E[e_0^2] = \frac{1-f}{n}(\beta_{2(y)} - 1)$, $E[e_1^2] = \frac{1-f}{n}(\beta_{2(x)} - 1)$ and $E[e_0e_1] = \frac{1-f}{n}(\lambda_{22} - 1)$.

The proposed estimator is given below:

$$\hat{S}_p^2 = s_y^2\left[\frac{S_x^2 + \alpha G}{s_x^2 + \alpha G}\right]$$

$$\Rightarrow \quad \hat{S}_p^2 = s_y^2(1+e_0)\left[\frac{S_x^2 + \alpha G}{s_x^2 + e_1 S_x^2 + \alpha G}\right]$$

$$\Rightarrow \quad \hat{S}_p^2 = \frac{S_y^2(1+e_0)}{(1+A_p e_1)}$$

where $A_p = \frac{S_x^2}{(S_x^2 + \alpha G)}$

$$\Rightarrow \hat{S}_p^2 = S_y^2(1+e_0)(1+A_p e_1)^{-1}$$

$$\Rightarrow \hat{S}_p^2 = S_y^2(1+e_0)(1 - A_p e_1 + A_p^2 e_1^2 - A_p^3 e_1^3 + ...)$$

Expanding and neglecting the terms more than 3rd order, we get

$$\hat{S}_p^2 = S_y^2 + S_y^2 e_0 - S_y^2 A_p e_1 - S_y^2 A_p e_0 e_1 + S_y^2 A_p^2 e_1^2 \tag{1}$$

$$\Rightarrow \hat{S}_p^2 - S_y^2 = S_y^2 e_0 - S_y^2 A_p e_1 - S_y^2 A_p e_0 e_1 + S_y^2 A_p^2 e_1^2 \tag{2}$$

By taking expectation on both sides of (1), we get

$$E\left(\hat{S}_p^2 - S_y^2\right) = S_y^2 E(e_0) - S_y^2 A_p E(e_1) - S_y^2 A_p E(e_0 e_1) + S_y^2 A_p^2 E(e_1^2)$$

$$BiasE\left(\hat{S}_p^2\right) = S_y^2 A_p^2 E(e_1^2) - S_y^2 A_p E(e_0 e_1)$$

$$BiasE\left(\hat{S}_p^2\right) = \gamma S_y^2 A_p [A_p(\beta_{2(x)} - 1) - (\lambda_{22} - 1)] \quad (3)$$

Squaring both sides of (2), neglecting the terms more than 2nd order and taking expectation, we get

$$E\left(\hat{S}_p^2 - S_y^2\right)^2 = S_y^4 E(e_0^2) - S_y^4 A_p^2 E(e_1^2) - 2S_y^4 A_p E(e_0 e_1)$$

$$MSE\left(\hat{S}_p^2\right) = \gamma S_y^4 [(\beta_{2(y)} - 1) + A_p^2(\beta_{2(x)} - 1) - 2A_p(\lambda_{22} - 1)]$$

4. EFFICIENCY CONDITIONS

In this section, we have derived the efficiency conditions of proposed estimators with other existing estimators under which proposed estimator are performing better than the existing estimators.

The bias and Mean square error of existing ratio type estimators up to the first order of approximation is given by

$$BiasE\left(\hat{S}_K^2\right) = \gamma S_y^2 R_K [R_K(\beta_{2(x)} - 1) - (\lambda_{22} - 1)] \quad (4)$$

$$MSE\left(\hat{S}_K^2\right) = \gamma S_y^4 [(\beta_{2(y)} - 1) + R_K^2(\beta_{2(x)} - 1) - 2R_K(\lambda_{22} - 1)] \quad (5)$$

Where R_K = Constant, $K = 1,2,3$

Bias, MSE and constant of proposed estimators is given by

$$Bias\left(\hat{S}_P^2\right) = \gamma S_y^2 R_P \left[R_P(\beta_{2x} - 1) - (\lambda_{22} - 1)\right] \quad (6)$$

$$MSE\left(\hat{S}_P^2\right) = \gamma S_y^4 \left[(\beta_{2y} - 1) + R_P^2(\beta_{2x} - 1) - 2R_P(\lambda_{22} - 1)\right] \quad (7)$$

R_p = Constant, $p = 1$

From Equation (4) and (6), we have

$$MSE\left(\hat{S}_P^2\right) \le MSE\left(\hat{S}_K^2\right) if \lambda_{22} \ge 1 + \frac{(R_P + R_K)(\beta_{2x} - 1)}{2}$$

$$MSE\left(\hat{S}_P^2\right) \le MSE\left(\hat{S}_K^2\right)$$

$$\gamma S_y^4 \left[(\beta_{2y} - 1) + R_P^2(\beta_{2x} - 1) - 2R_P(\lambda_{22} - 1)\right] \le \gamma S_y^4 \left[(\beta_{2y} - 1) + R_K^2(\beta_{2x} - 1) - 2R_K(\lambda_{22} - 1)\right] \quad (8)$$

$$\Rightarrow \left[(\beta_{2y} - 1) + R_P^2(\beta_{2x} - 1) - 2R_P(\lambda_{22} - 1)\right] \le \left[(\beta_{2y} - 1) + R_K^2(\beta_{2x} - 1) - 2R_K(\lambda_{22} - 1)\right] \quad (9)$$

$$\Rightarrow \left[1 + R_P^2(\beta_{2x} - 1) - 2R_P(\lambda_{22} - 1)\right] \le \left[1 + R_K^2(\beta_{2x} - 1) - 2R_K(\lambda_{22} - 1)\right] \quad (10)$$

$$\Rightarrow (\beta_{2x} - 1)\left(R_P^2 - R_K^2\right) \left[-2R_P(\lambda_{22} - 1)\right] \le \left[-2R_K(\lambda_{22} - 1)\right] \quad (11)$$

$$\Rightarrow (\beta_{2x} - 1)\left(R_P^2 - R_K^2\right) \left[-2(\lambda_{22} - 1)(R_P - R_K)\right] \le 0 \quad (12)$$

$$\Rightarrow (\beta_{2x} - 1)\left(R_P^2 - R_K^2\right) \le \left[2(\lambda_{22} - 1)(R_P - R_K)\right] \quad (13)$$

$$\Rightarrow (\beta_{2x} - 1) \le \frac{2(\lambda_{22} - 1)(R_P - R_K)}{\left(R_P^2 - R_K^2\right)} \quad (14)$$

$$\Rightarrow (\beta_{2x} - 1) \le \frac{2(\lambda_{22} - 1)(R_P - R_K)}{(R_P - R_K)(R_P + R_K)} \quad (15)$$

$$\Rightarrow (\beta_{2x} - 1)\ (R_P + R_K) \le 2(\lambda_{22} - 1)$$

By solving equation (15), we get

$$MSE\left(\hat{S}_P^2\right) \le MSE\left(\hat{S}_K^2\right) if \lambda_{22} \ge 1 + \frac{(R_P + R_K)(\beta_{2x} - 1)}{2}$$

5. NUMERICAL ILLUSTRATION

We use the data of Murthy [5] page 228 in which fixed capital is denoted by X (auxiliary variable) and output of 80 factories are denoted by Y(study variable).we apply the proposed and existing estimators to this data set and the data statistics is given below:

Table 1.Bias and mean square error of the existing and the proposed estimators

Estimators	Bias	Mean square error
Isaki	10.8762	3925.1622
Kadilar & Cingi	10.4399	3850.1552
Subramani & Kumarapandiyan	6.1235	3180.7740
Proposed	4.8896	2452.3107

Table 2. Percent relative efficiency of proposed estimators with existing estimators

Estimators	Isaki	Kadilar and Cingi	Subramani and Kumarapandiyan
Proposed	160.0597	157.0011	129.7051

N=80 ,Sx=8.4542, n=20, Cx=0.7507, $\overline{X} =$ 11.2624, $\beta_{2(x)} =$ 2.8664, $\overline{Y} =$ 51.8264, $\beta_{2(y)} =$ 2.8664, $\rho =$ 0.9413, $\beta_{1(x)} =$ 1.05, $\lambda_{22} =$ 2.2209, S_y=18.3569, Md=7.5750,Q1=9.318, C_y=0.3542, G = 9.0408 D= 8.0138 S_{pw} = 7.9136

6. CONCLUSION

From the above study we revealed that our suggested estimator performed better than the existing estimators in terms of mean square error and bias since they are significant lower than the corresponding values of the existing estimators.'' too much lower than existing estimators.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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